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ABSTRACT

This document describes a mathematics course that uses the computer to solve mathematics problems. It was developed to be used with students who have completed at least one year of general mathematics or are not achieving success in the traditional mathematics program. The course is intended to review, reinforce, and extend concepts included in the secondary school mathematics curricula. The major focus of the course is on problem solving. The teaching of computer programming is intended to provide a model of a problem solving process in which a problem is broken down into its component parts. The book contains 61 lessons, some of which are laboratory sessions, which use the computer and the process of computer programming to help students understand: (1) flowcharts; (2) rational numbers; (3) algebraic equations; (4) geometric principles; (5) temperature conversion; (6) number systems; (7) factoring; (8) prime numbers; (9) square root; (10) rounding; (11) the Pythagorean theorem; (12) probability; and (13) statistics. (TW)

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7-10-87

FOREWORD

Computer Mathematics: An Introduction Part II is a mathematics course that uses the computer to solve mathematics problems. Students are given opportunities to explore various branches of mathematics. They are then taught how to develop computer solutions to problems and extensions of problems in the different areas of mathematics.

These computer lessons are designed not only to make the student computer literate but also to:

a. increase his/her motivation to study mathematics,
b. improve his/her ability to conceptualize,
c. improve his/her problem solving skills,

all leading to an improvement in performance in mathematics and in other subject areas.

We believe that by implementing this course concomitant benefits will be:

a. increasing the school's holding power and
b. providing another option to meet the New York State Regents Action Plan requirements for a second year of mathematics credit for a local diploma.

This course has been approved by the New York State Education Department's Bureau of Mathematics for a second year of credit in mathematics for a local diploma.

Charlotte Frank

Charlotte Frank
Executive Director
Division of Curriculum and Instruction

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INTRODUCTION

Computer Mathematics is a course designed to engage students in using the computer to solve mathematical problems. It has been prepared to be used with students who have completed a year of general mathematics or for those who are not meeting success in the more traditional mathematics program. The purpose of this course is to review, reinforce and extend concepts from the secondary school mathematics curricula. We have found that by directing students' interest in computers toward the study of mathematics, they not only become computer literate but also better mathematics students.

The focus of this course is problem solving. The process of problem solving, as treated in this curriculum, can be compared to that which a tutor must undergo in teaching someone else. In explaining how a problem is solved, the tutor's thought processes become crystalized; thus, he/she develops a deeper understanding of how to solve a problem. When we program a computer, we must first break the problem down into its component parts which we then translate into a computer language. In this course, students are required to participate in a similar process. This activity evolves genuine problem solving skills, applicable to many situations.

ACKNOWLEDGMENTS

This curriculum guide, Computer Mathematics: An Introduction (Part II) was prepared by the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

Morris Freedman, Deputy Executive Director of the Office of Curriculum Development and Support, provided overall supervision of this project.

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Kent Beaty designed the cover.

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Lesson: 1

AIM: How do we develop a flowchart that will arrange three rational numbers in ascending order?

PERFORMANCE OBJECTIVES: The student will be able to...

1. order a set of rational numbers;
2. use <, >, or = to describe the order of rational numbers;
3. convert a written plan into a flowchart.

VOCABULARY: order, is less than, is greater than, branch, ascending order, descending order, permutation

MOTIVATION: In each triple, which represents the largest number?

1) .3, .30, .300

2) .3, .03, .003

DEVELOPMENT:

1. Elicit responses to the motivation examples. Review the order on the number line and the appropriate symbolism. Assign Applications examples 1 and 2.
2. Present the problem below for a computer solution.

   Problem: Enter three different numbers. Have the computer arrange them in ascending order.
3. Remind the students that when solving problems, it is often a good technique to start with a simpler problem of the same type in order to discover a method.
   Ask, "How can we get the computer to arrange two numbers in ascending order?"
4. Elicit a plan for arranging two numbers in ascending order as follows.

   Plan

   1) Enter two different numbers: X,Y.
   2) Check: Is X < Y?
      a) If yes, then print X,Y. Go to 1.
      b) If no, continue.

```
   3)   Print Y, X.
   4)   Go to 1.
   5)   End.
```

5. Ask, "How can we extend this plan to three numbers?"

6. Elicit the plan for three numbers as follows. Use three test values in various orders to help the students develop the plan.

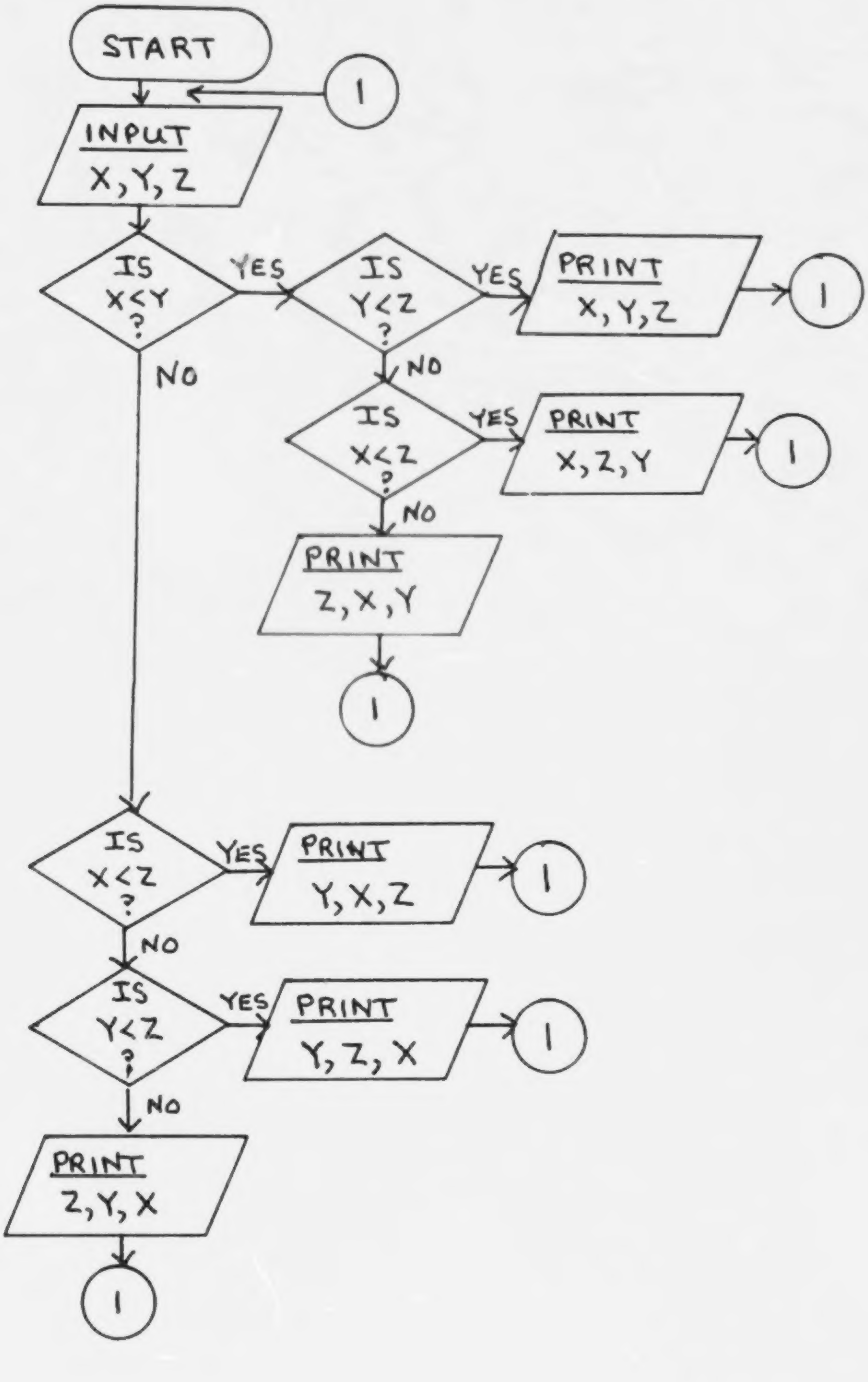
Plan

```
   1)    Enter any three numbers:  X,Y,Z.
   2)    Check:  Is X < Y?
                 (The students should be made aware that
                 printing cannot be done until the order of
                 all three numbers is determined.)
         a)  If yes, check:  Is Y < Z?
               i)  If yes, print X,Y,Z.  Go to 1.
              ii)  If no, check: Is X<Z?
                                        a) If yes, print
                                           X,Z,Y. Go to 1.
                                        b) If no, print Z,X,Y.
                                           Go to 1.
         b)  If no, ...
```

7. The students will realize at this point that the plan is complex. Ask, "Is there any way we can illustrate this plan so that it will be easier to follow and develop?"

8. Elicit the flowchart which follows.
Review the symbols used in a flowchart and develop the flowchart from the part of the plan already done. Ask the students to complete the flowchart; for those unable, help them at their seats. To further assist the students in developing the flowchart, ask them to mark the letters X,Y and Z on three small sheets of paper (or index cards). As the checks are made, the students should rearrange the marked papers so that when ready to write each PRINT statement, the papers will be appropriately arranged. When the flowcharts are completed, ask, "How many different arrangements (permutations) are there for the letters X,Y and Z? Are all of these arrangements included in your flowchart?"

Flowchart


START
1
INPUT
X, Y, Z
IS X<Y ?
YES
NO
IS Y<Z ?
YES
PRINT
X, Y, Z
1
NO
IS X<Z ?
YES
PRINT
X, Z, Y
1
NO
PRINT
Z, X, Y
1
IS X<Z ?
YES
PRINT
Y, X, Z
1
NO
IS Y<Z ?
YES
PRINT
Y, Z, X
1
NO
PRINT
Z, Y, X
1

9. Ask, "Does the flowchart 'work' if the numbers are not all different?" Elicit a triple of numbers that can be used to test the students' prediction.

10. Assign Applications examples 3 and 4.

APPLICATIONS:

1. Use <, >, or = to describe the order of each pair of numbers.
   - a) 5.0,.50
   - b) .23,.4
   - c) 12.2,1.22
   - d) .750,.7500
   - e) 0.340,.034

2. Arrange each triple of numbers in ascending order.
   - a) .3,.03,.23
   - b) 0.7,.07,7.0
   - c) 0.33,.033,.330
   - d) 1.5,1.500,1.50

3. Trace the flowchart developed in this lesson using the four triples of values in 2 above.

4. Trace the flowchart choosing your own values.

SUMMARY:

1. How do we determine the order of rational numbers?

2. Why does our flowchart have many branches?

HOMEWORK:

1. Use <, >, or = to describe the order of each pair of numbers.
   - a) .575,.3
   - b) .001,0.01
   - c) 7.05,7.50
   - d) 60.5,60.500

2. Arrange each triple of numbers in ascending order by tracing the flowchart developed in class.
   - a) .002,2.0,.20
   - b) 23.5,25,5.30
   - c) .85,.805,8.05
   - d) 99.0,9.9,0.99

3. Arrange each triple of numbers in descending order.
   - a) 1.5,0.5,3.0
   - b) .001,.01,0.1
   - c) 5.2,5.200,8.25
   - d) 5,3.27,18.45

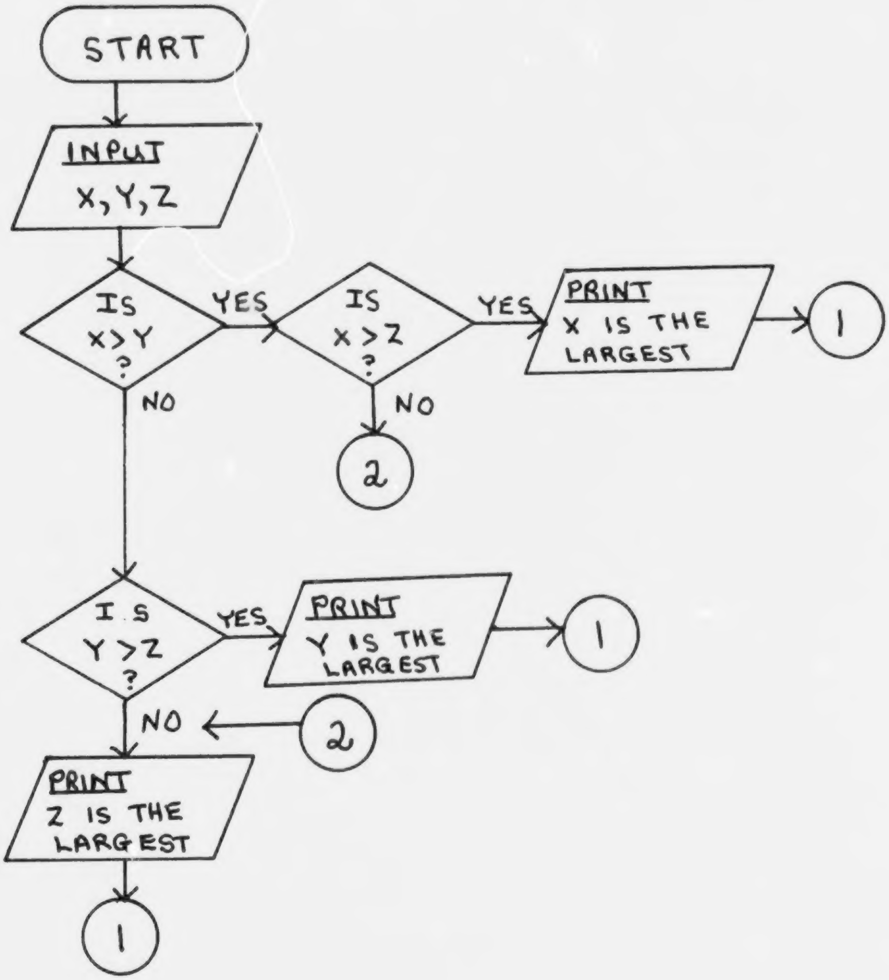
4. Develop a flowchart that will solve the following problem.

Problem: Enter three numbers. Have the computer find and display the largest number.

HOMEWORK SOLUTIONS:

1. a) .575 > .3
   b) .001 < 0.01
   c) 7.05 < 7.50
   d) 60.5 = 60.500
2. a) .002,.20,2.0
   b) 5.30,23.5,25
   c) .805,.85,8.05
   d) 0.99,9.9,99.0
3. a) 3.0,1.5,0.5
   b) 0.1,.01,.001
   c) 8.25,5.2,5.200 (or 8.25,5.200,5.2)
   d) 18.45,5,3.27
4. See the next page.

Flowchart


START
INPUT
X, Y, Z
IS
X>Y
?
YES
NO
IS
X > Z
?
YES
NO
PRINT
X IS THE LARGEST
1
2
IS
Y > Z
?
YES
NO
PRINT
Y IS THE LARGEST
1
2
PRINT
Z IS THE LARGEST
1

Lesson: 2L

AIM: How do we design a planned output and code a program to arrange three rational numbers in ascending order?

PERFORMANCE OBJECTIVES: The student will be able to...

1. design a planned output;
2. write a program using the flowchart and planned output;
3. test and debug the program.

VOCABULARY: planned output, title, prompt, debug, ascending order, descending order, permutation

MOTIVATION: Use the flowchart developed in the previous lesson to arrange each triple of rational numbers in ascending order.

1) .055,.005,.105

2) 4.69,4.23,4.058

DEVELOPMENT:

1. Review the flowchart and order relations using the responses from the motivation.
2. Ask, "To write a program, is there any other planning required in addition to the flowchart?"
3. Elicit a planned output as follows. Remind the students that a title, prompt, and written solution are needed. Allow the students to develop their own planned outputs and present them to the class.

Planned Output

```
ARRANGING NUMBERS IN ASCENDING ORDER
ENTER THREE NUMBERS.
? .055,.005,.105
THE NUMBERS IN ASCENDING ORDER ARE .005,.055,.105
ENTER THREE NUMBERS.
? 4.69,4.23,4.058
THE NUMBERS IN ASCENDING ORDER ARE 4.058,4.23,4.69
ENTER THREE NUMBERS.
?
```

4. Elicit the program which follows. Remind the students that when coding decisions, we program each NO branch before the YES branch and then proceed back up to the top to code each YES branch. It is recommended that this program be developed with the students at the chalkboard either entirely or in branches.

Program

```
10     PRINT "ARRANGING NUMBERS IN ASCENDING ORDER"
20     PRINT "ENTER THREE NUMBERS."
30     INPUT X,Y,Z
40     IF X < Y THEN 90
50     IF X < Z THEN 130
60     IF Y < Z THEN 150
70     PRINT "THE NUMBERS IN ASCENDING ORDER ARE";Z;",";Y;",";X
80     GOTO 20
90     IF Y < Z THEN 170
100    IF X < Z THEN 190
110    PRINT "THE NUMBERS IN ASCENDING ORDER ARE";Z;",";X;",";Y
120    GOTO 20
130    PRINT "THE NUMBERS IN ASCENDING ORDER ARE";Y;",";X;","Z
140    GOTO 20
150    PRINT "THE NUMBERS IN ASCENDING ORDER ARE";Y;",";Z;",";X
160    GOTO 20
170    PRINT "THE NUMBERS IN ASCENDING ORDER ARE";X;",";Y;",";Z
180    GOTO 20
190    PRINT "THE NUMBERS IN ASCENDING ORDER ARE";X;",";Z;",";Y
200    GOTO 20
210    END
```

(Note to the teacher: You might ask the students to suggest more efficient ways of printing the message THE NUMBERS IN ASCENDING ORDER ARE; e.g,

```
35     PRINT "THE NUMBERS IN ASCENDING ORDER ARE";
70     PRINT Z;",";Y;",";X
110    PRINT Z;",";X;",";Y
130    PRINT Y;",";X;",";Z
150    PRINT Y;",";Z;",";X
170    PRINT X;",";Y;",";Z
190    PRINT X;",";Z;",";Y)
```

5. Ask the students to enter, run, and test the program with the appropriate values from the previous homework.

6. Assign the Application example.

APPLICATION:

Ask the students to revise the program so that it will arrange the numbers in descending order.

SUMMARY:

What procedure should be followed when coding a program with multiple branches?

HOMEWORK:

1. Design an output and code a program (using the flowchart developed in the previous homework, question #4) to have the computer select the largest of three numbers.

2. (Optional) For the following problem, complete the instructions.
   a) Develop a flowchart for a computer solution.
   b) List all twenty-four arrangements (permutations) of the four numbers .1,.2,.3 and .4. Trace your flowchart using each of the arrangements.
   c) Design an output.
   d) Code a program.

Problem: Enter four numbers. Have the computer find and display the smallest number.

(Allow the students at least one week to submit this assignment for extra credit.)

HOMEWORK SOLUTIONS:

1. Planned Output

```
SELECTING THE LARGEST NUMBER
ENTER THREE NUMBERS.
? .1,.2,.3
THE LARGEST NUMBER IS .3
ENTER THREE NUMBERS.
? .2,.1,.3
THE LARGEST NUMBER IS .3
ENTER THREE NUMBERS.
? .2,.3,.1
THE LARGEST NUMBER IS .3
ENTER THREE NUMBERS.
? .1,.3,.2
THE LARGEST NUMBER IS .3
ENTER THREE NUMBERS.
? .3,.2,.1
THE LARGEST NUMBER IS .3
ENTER THREE NUMBERS.
? .3,.1,.2
THE LARGEST NUMBER IS .3
ENTER THREE NUMBERS.
?
```

Program

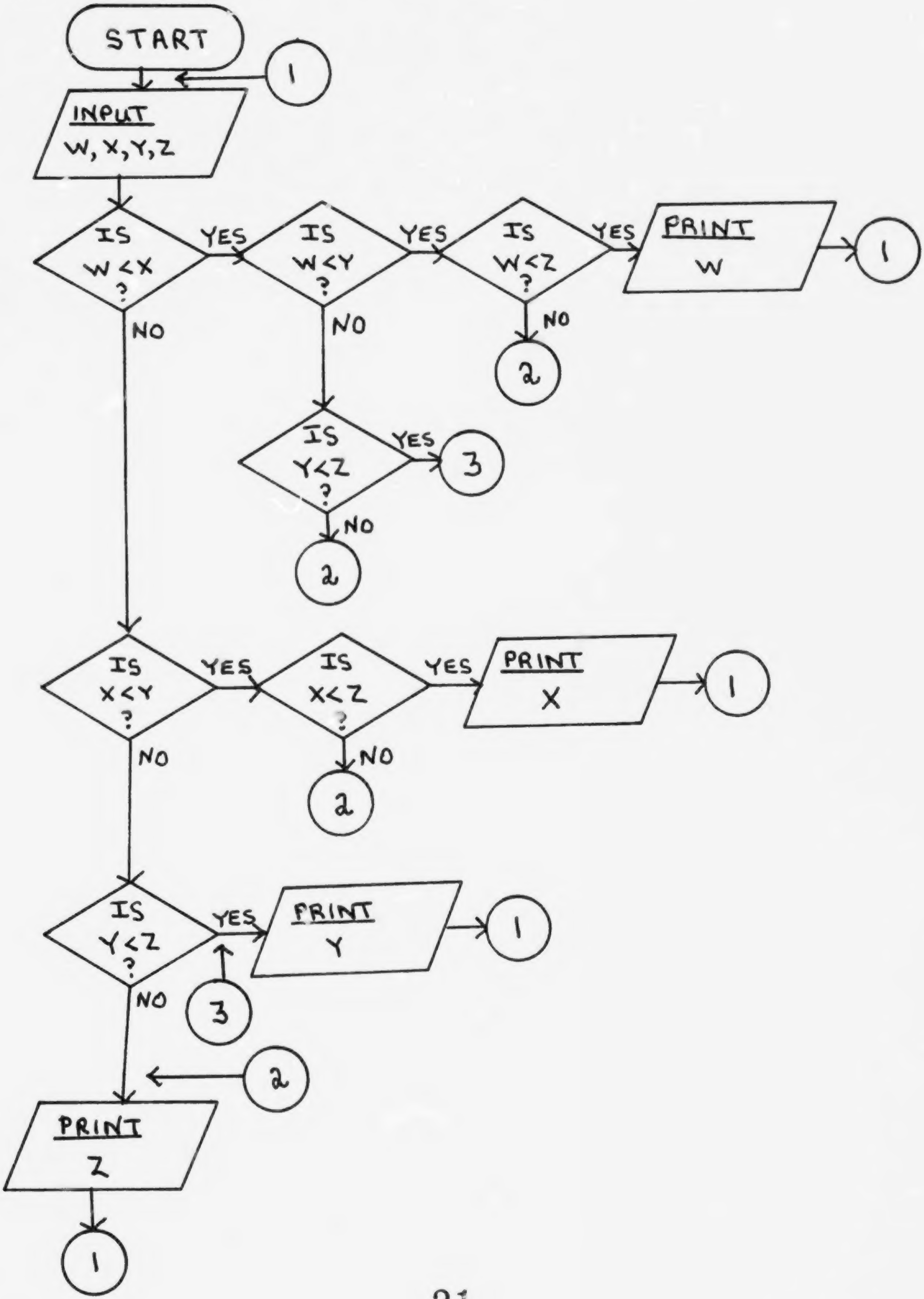
```
10  PRINT "SELECTING THE LARGEST NUMBER"
20  PRINT "ENTER THREE NUMBERS."
30  INPUT X,Y,Z
40  IF X > Y THEN 80
50  IF Y > Z THEN 100
60  PRINT "THE LARGEST NUMBER IS";Z
70  GOTO 20
80  IF X > Z THEN 120
90  GOTO 60
100 PRINT "THE LARGEST NUMBER IS";Y
110 GOTO 20
120 PRINT "THE LARGEST NUMBER IS";X
130 GOTO 20
140 END
```

(Note to the teacher: You might ask the students to suggest more efficient ways of printing the message THE LARGEST NUMBER IS; e.g.,

```
35  PRINT "THE LARGEST NUMBER IS";
60  PRINT Z
100 PRINT Y
120 PRINT X)
```

2. See the next page.

2a) Flowchart


START
1
INPUT
W, X, Y, Z
IS W<X ?
YES
NO
IS W<Y ?
YES
NO
IS W<Z ?
YES
NO
PRINT
W
1
2
IS Y<Z ?
YES
NO
3
2
IS X<Y ?
YES
NO
IS X<Z ?
YES
NO
PRINT
X
1
2
IS Y<Z ?
YES
NO
PRINT
Y
1
3
2
PRINT
Z
1

2b)

.1,.2,.3,.4
.1,.2,.4,.3
.1,.3,.4,.2
.1,.3,.2,.4
.1,.4,.2,.3
.1,.4,.3,.2

.2,.1,.3,.4
.2,.1,.4,.3
.2,.3,.1,.4
.2,.3,.4,.1
.2,.4,.1,.3
.2,.4,.3,.1

.3,.1,.2,.4
.3,.1,.4,.2
.3,.2,.1,.4
.3,.2,.4,.1
.3,.4,.1,.2
.3,.4,.2,.1

.4,.1,.2,.3
.4,.1,.3,.2
.4,.2,.1,.3
.4,.2,.3,.1
.4,.3,.2,.1
.4,.3,.1,.2

2c) Planned Output

```
SELECTING THE SMALLEST NUMBER
ENTER FOUR NUMBERS.
? .1,.2,.3,.4
THE SMALLEST NUMBER IS .1
ENTER FOUR NUMBERS.
?
```

2d) Program

```
10  PRINT "SELECTING THE SMALLEST NUMBER"
20  PRINT "ENTER FOUR NUMBERS."
30  INPUT W,X,Y,Z
40  PRINT "THE SMALLEST NUMBER IS";
50  IF W < X THEN 100
60  IF X < Y THEN 130
70  IF Y < Z THEN 150
80  PRINT Z
90  GOTO 20
100 IF W < Y THEN 170
110 IF Y < Z THEN 150
120 GOTO 80
130 IF X < Z THEN 210
140 GOTO 80
150 PRINT Y
160 GOTO 20
170 IF W < Z THEN 190
180 GOTO 80
190 PRINT W
200 GOTO 20
210 PRINT X
220 GOTO 20
230 END
```

Lesson: 3

AIM: How do we solve for x, all equations of the form ax+b=c?

PERFORMANCE OBJECTIVES: The student will be able to...

1. use inverse operations to solve equations involving two operations;
2. check the solutions by substitution.

VOCABULARY: inverse operation, equivalent equation, substitution

MOTIVATION: Solve and check each equation.

1)	$x+18=35$	3)	$4x=28$
2)	$y-12=15$	4)	$w/3=2$

DEVELOPMENT:

1. Elicit responses to the motivation problems. Review the procedures for solving equations using the inverse operations. (Part I, Lesson 36)
2. Present the equation $3x + 5 = 29$ for an algebraic solution. Ask, "What was done to the number represented by x in order to obtain 29? How can we use the inverse operations to solve for x?"
3. Guide the students to "undo" the operations in the reverse order. Elicit the procedure as follows.
   a) Ask, "What was the last operation done? How do we 'undo' the addition of 5?"
   Subtract 5 from both sides:
   $$\begin{array}{r} 3x+\cancel{5}=29 \\ \underline{\cancel{-5}\ \ -5} \\ 3x\ \ \ =24 \end{array}$$
   b) Ask, "How do we 'undo' the multiplication by 3?"
   Divide both sides by 3:
   $$\frac{3x}{3} = \frac{24}{3}$$
   $$x = 8$$

c) Ask, "How do we know that 8 is the correct value for x?" Check by substitution:

$$\begin{aligned} 3x+5&=29 \\ 3(8)+5&\overset{?}{=}29 \\ 24+5&\overset{?}{=}29 \\ 29&=29 \end{aligned}$$

4. Ask, "Could we have 'undone' the multiplication first?" Guide the students through the procedure as follows.

   a) Divide both sides by 3:

   $$\frac{3x+5}{3} = \frac{29}{3}$$

   Stress that all three terms must be divided by 3.

   $$\frac{3x}{3} + \frac{5}{3} = \frac{29}{3}$$

   $$x + \frac{5}{3} = \frac{29}{3}$$

   b) Ask, "How do we 'undo' the addition of $\frac{5}{3}$?"

   Subtract $\frac{5}{3}$ from both sides:

   $$\begin{aligned} x + \cancel{\tfrac{5}{3}} &= \tfrac{29}{3} \\ \cancel{-\tfrac{5}{3}} &\quad -\tfrac{5}{3} \\ \hline x &= \tfrac{24}{3} \text{ or } x = 8 \end{aligned}$$

5. Ask, "Which procedure do you prefer? Why?" Stress that to "undo" the operations, the reverse of the usual order of operations is preferable.

6. Ask the students to complete the Applications problems.

APPLICATIONS:

Solve and check the following equations.

1) $5x+12=32$
2) $15=2x+5$
3) $3y-10=17$
4) $22=3z-2$
5) $x/3+2=7$
6) $y/5-11=9$
7) $20=w/4+8$
8) $2.5x-0.1=4.9$

SUMMARY:

What order do we use in solving equations with two operations?

HOMEWORK:

Solve and check each equation.

1) $7x+4=18$
2) $25=x/2-6$
3) $12y-45=129$
4) $w/7-3=12$
5) $3.2z+1.8=4.36$
6) $93=34x-145$
7) $v/1.5+.9=2.1$
8) $6n+3=25$

HOMEWORK SOLUTIONS:

1) $x=2$
2) $62=x$
3) $y=14.5$
4) $w=105$
5) $z=0.8$
6) $7=x$
7) $v=1.8$
8) $n=3.666...$or $n=3.\overline{6}$ or $n=3\frac{3}{5}$

Lesson: 4

AIM: How do we solve verbal problems that lead to equations of the form $ax+b=c$?

PERFORMANCE OBJECTIVES: The student will be able to...

1. translate a verbal problem into an equation;
2. solve and check the equation;
3. determine the solution to the problem and check it.

VOCABULARY: more than, less than, increase, decrease, diminish, exceeded by, times, of, twice, ...

MOTIVATION: Translate each phrase into mathematical symbols.

1. a number increased by ten
2. six times a number
3. three less than a number
4. a number exceeded by three
5. twice a number increased by seven

DEVELOPMENT:

1. Elicit responses to the motivation examples. Review the translation of the words into mathematical symbols. (Part I, Lesson 37)
2. Write the following problem on the chalkboard.

   Problem: Twice a number decreased by seven is twenty-five. Find the number.

3. Ask, "How can we solve this problem algebraically?"
4. Elicit the following.
   a) Read the problem and guess the solution.
   b) Determine what is unknown and represent it as a variable. (Let x = the number)
   c) Translate the words into mathematical symbols. ($2x-7=25$)

   d) Solve and check the equation. ($x=16$)
   e) Determine and write the solution. (Sometimes, the value of x is not the solution.)
   f) Check the solution with respect to the words of the problem. (How does it compare to your guess? Is it reasonable?)

5. Complete the first Applications problem by eliciting responses from the students and reinforcing 4a-4f.

6. Assign Applications problems 2 and 3.

APPLICATIONS:

Solve algebraically and check.

1. The Knicks won 56 games. They won 12 more than half of the number they lost. Find the number of losses.

2. The cost of a long distance call from New York to New Jersey is $.45 for the first 3 minutes and $.25 for each additional minute. Jennifer, who lives in New York, has $2.20. How many minutes can she speak to her friend Maria in New Jersey?

3. Leonard opens a savings account with $25 and can save $15 each week. How many weeks would it take for him to save $160?

SUMMARY:

What are the steps that we should follow in solving verbal problems algebraically?

HOMEWORK:

Solve each problem algebraically and then check it.

1. A salesman receives a salary of $125 plus $28 for each sale he makes. How many sales would it take to earn $321?

2. Sammy's age is 3 years less than half his sister's age. If Sammy is 12 years old, how old is his sister?

3. One-third of a number diminished by 12 is 8. What is the number?

4. Mr. James wants to rent a car for the day. The cost is $12 plus $.07 per mile. How many miles can Mr. James drive if he spends $26.77 on the car rental?

5. One hundred fifty students attended a school dance. This number of students is 32 less than $\frac{2}{5}$ of the students in the school. How many students are there in the school?

HOMEWORK SOLUTIONS:

1. It would take 7 sales.
2. Sammy's sister is 30 years old.
3. The number is 60.
4. Mr. James can drive 211 miles on the car rental.
5. There are 455 students in the school.

Stress that the students should have followed the procedure outlined in 4a-4f (Development) when solving and checking the five verbal problems.

LESSON: 5L

AIM: How do we write a program to solve for X, all equations of the form AX+B=C?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop an algorithm for solving AX+B=C for X;
2. incorporate the algorithm X=(C-B)/A into a plan for a computer solution;
3. develop a flowchart;
4. design a planned output;
5. write a program;
6. have the computer skip lines in a BASIC program.

VOCABULARY: equation, variable, coefficient, constant, solution

MOTIVATION: Solve algebraically.

Nine more than four times a number is twenty-five. What is the number?

DEVELOPMENT:

1. Elicit an algebraic solution to the motivation problem.
2. Ask, "How can we develop a computer solution for all equations of the form used in our problem?"
   Remind the students that to develop a computer solution requires a(n) analysis, plan, flowchart and planned output.
3. Elicit the analysis as follows.
   Analysis
   a) Ask, "How can we generalize equations of the type used in the motivation?" (AX+B=C)
   b) Ask, "How can we express this equation in the form 'X='?"

   On the chalkboard, solve both 4X+9=25 and AX+B=C using a parallel development.

$$\begin{array}{rcl} \underline{\text{ORIGINAL EQUATION}} & & \\ 4X+\cancel{9} & = & 25 \\ \cancel{-9} & & -9 \\ \hline \frac{4X}{4} & = & \frac{16}{4} \\ & X=4 & \end{array} \qquad \begin{array}{rcl} \underline{\text{GENERAL EQUATION}} & & \\ AX+\cancel{B} & = & C \\ \cancel{-B} & & -B \\ \hline \frac{AX}{A} & = & \frac{C-B}{A} \\ X & = & \frac{C-B}{A} \end{array}$$

c) Say to the students that X=(C-B)/A is the algorithm used to solve for X any equation of the type AX+B=C.

Ask, "Why are the parentheses necessary?"

4. Ask the students to complete Applications example 1.

5. Ask, "How can we incorporate this algorithm into a plan?" Elicit the plan which follows.

Plan

1) Enter the values of A,B, and C.

*2) Check: Is A=0?

  a) If yes, print 'A' CANNOT EQUAL 0. Go to 1.

  b) If no, continue.

3) Calculate: X=(C-B)/A.

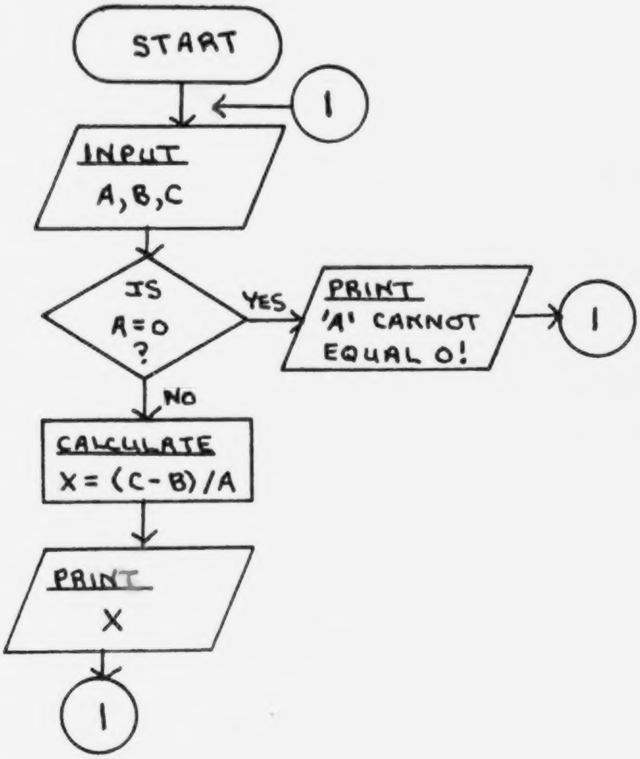
4) Print the solution for X.

5) Go to 1.

*Remind the students that division by zero is undefined and will cause an error message.

6. Ask the students to flowchart the plan as follows.

Flowchart


START
1
INPUT
A,B,C
IS
A=0
?
YES
PRINT
'A' CANNOT
EQUAL 0!
1
NO
CALCULATE
X=(C-B)/A
PRINT
X
1

7. Ask the students to design a planned output as follows.

Planned Output

```
SOLVING EQUATIONS OF THE FORM AX+B=C
(Skip 2 lines.)
ENTER THE CONSTANTS A,B, AND C.
? 4,9,25
(Skip 1 line.)
X=4
(Skip 2 lines.)
ENTER THE CONSTANTS A,B, AND C.
? 0,6,18
(Skip 1 line.)
'A' CANNOT EQUAL ZERO!
(Skip 2 lines.)
ENTER THE CONSTANTS A,B, AND C.
?
```

Ask, "How do we get the computer to skip lines?"

8. Ask the students to code and run the program which follows.

Program

```
10  PRINT "SOLVING EQUATIONS OF THE FORM AX+B=C"
20  PRINT
30  PRINT
40  PRINT "ENTER THE CONSTANTS A,B, AND C."
50  INPUT A,B,C
60  PRINT
70  IF A=0 THEN GOTO 110
80  X=(C-B)/A
90  PRINT "X=";X
100 GOTO 20
110 PRINT "'A' CANNOT EQUAL ZERO!"
120 GOTO 20
130 END
```

9. Assign Applications problems 2,3,4, and 5.

APPLICATIONS:

1. Use the algorithm X=(C-B)/A to solve the following equations.
   - a) 5X+12=47
   - b) 2X+1.5=9.5
   - c) 2.2X+3.4=12.2
   - d) 1040=15X+200
2. Revise the output and program developed in this lesson so that the equation is displayed as well as the solution.
3. Trace your program for the following values of A,B, and C.
   - a) 5,45,100
   - b) 0,20,15
   - c) 2,20,25
   - d) 10,14.5,54.5
   - e) 2.3,0.8,2.18
4. Revise your program so that it will solve for X all equations of the form AX-B=C.

5. Use the revised program to solve the following equations.
   - a) 3X-7=29
   - c) 4X-4.5=19.5
   - b) 6X-12=30
   - d) .6X-4.04=8.08

SUMMARY:

What is the advantage of using the algorithm when solving equations for X?

HOMEWORK:

1. Use the appropriate program that was developed in class, to solve each of these equations.
   - a) 12X+132=192
   - c) 5.5=5X+.5
   - b) 3X-18=24
   - d) 5.2X-1.8=6.52
2. Develop an algorithm which can be used to solve equations of the form X/A+B=C for X.
3. Use the algorithm developed in problem 2 to solve the following equations.
   - a) X/2+6=18
   - c) X/3+2.5=13.2
   - b) X/8+9=20
   - d) X/0.4+12.6=28.6
4. (For Extra Credit) For the following problem, complete the instructions.
   - a) Write an analysis which includes the development of an algorithm.
   - b) Write a plan for a computer solution.
   - c) Develop a flowchart.
   - d) Design a planned output.
   - e) Code a program.

Problem: Have the computer solve for X all equations of the form X/A+B=C.

(Allow the students at least one week to prepare this assignment for extra credit.)

HOMEWORK SOLUTIONS:

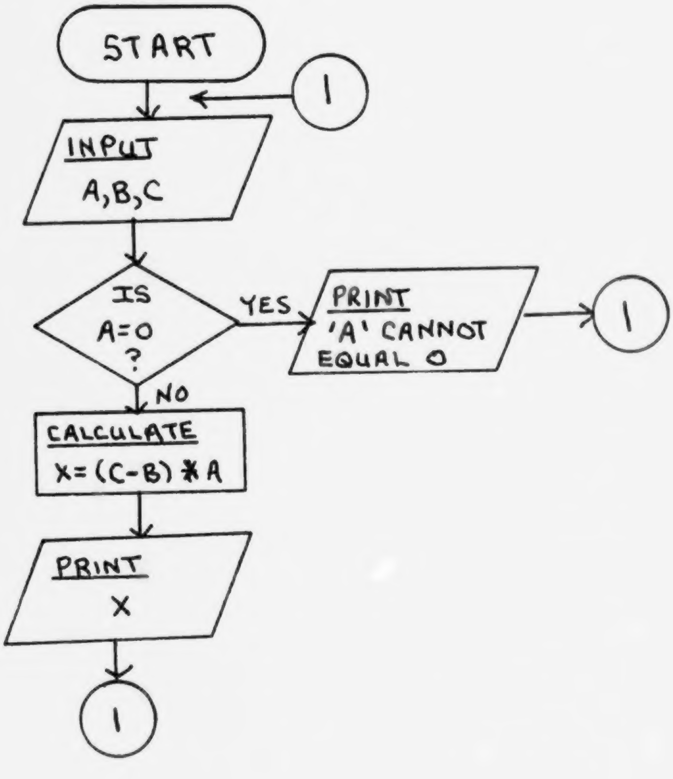
1. a) X=5 b) X=14 c) X=1 d) X=1.6
2. X=(C-B)*A
3. a) X=24 b) X=88 c) X=32.1 d) X=6.4

4a) The algorithm becomes X=(C-B)*A where memory cells A,B and C hold the values of the constants entered and memory cell X holds the solution.

4b) Plan

1. Enter the values of A,B, and C.
2. Check: Is A=0?
    a) If yes, print 'A' CANNOT EQUAL ZERO! Go to 1.
    b) If no, continue.
3. Calculate: X = (C-B)*A.
4. Print the solution for X.
5. Go to 1.

4c) Flowchart


START
1
INPUT
A,B,C
IS
A=0
?
YES
PRINT
'A' CANNOT
EQUAL 0
1
NO
CALCULATE
X=(C-B)*A
PRINT
X
1

4d) Planned Output

```
SOLVING EQUATIONS OF THE FORM X/A+B=C
(Skip 2 lines.)
ENTER THE CONSTANTS A,B, AND C.
? 2,6,18
(Skip 1 line.)
X=24
(Skip 2 lines.)
ENTER THE CONSTANTS A,B, AND C.
? 0,9,15
(Skip 1 line.)
'A' CANNOT EQUAL ZERO!
(Skip 2 lines.)
ENTER THE CONSTANTS A,B, AND C.
?
```

4e) Program

```
10  PRINT "SOLVING EQUATIONS OF THE FORM X/A+B=C"
20  PRINT
30  PRINT
40  PRINT "ENTER THE CONSTANTS A,B, AND C."
50  INPUT A,B,C
60  PRINT
70  IF A=0 THEN GOTO 110
80  X=(C-B)*A
90  PRINT "X=";X
100 GOTO 20
110 PRINT "'A' CANNOT EQUAL ZERO!"
120 GOTO 20
130 END
```

Lesson: 6

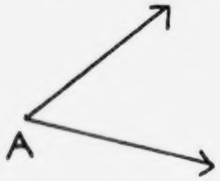
AIM: How do we develop a flowchart to classify angles?

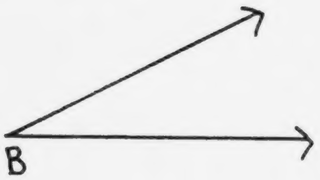
PERFORMANCE OBJECTIVES: The student will be able to...

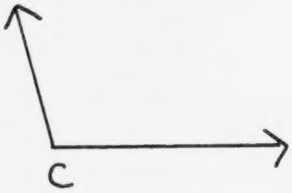
1. classify angles by measure;
2. develop a flowchart to determine the classification of an angle, given its measure.

VOCABULARY: angle, measure, acute angle, right angle, obtuse angle, straight angle, reflex angle

MOTIVATION: Which of the following angles is the greatest?







DEVELOPMENT:

1. Elicit responses to the motivation question.
   a) Review the definition of angle.
      Definition: An angle is the union of two rays (sides) at a commmon endpoint (vertex).
   b) Review the meaning of the measure of an angle and the definition of degree as a standard unit of angle measure.
      Definition: The measure of an angle is the amount (in a specified unit) of rotation of the rays.
      Definition: A degree is a unit of measure which is 1/360 of a complete rotation.
2. Ask the students to complete Applications example 1. As you elicit responses to the examples, ask, "How do you know?"
3. With reference to Applications example 1, define the classifications of angles.
4. Ask the students to complete Applications example 2.

5. Present the problem below for a computer solution.

Problem: Enter the measure of an angle. Have the computer determine the classification of the angle.

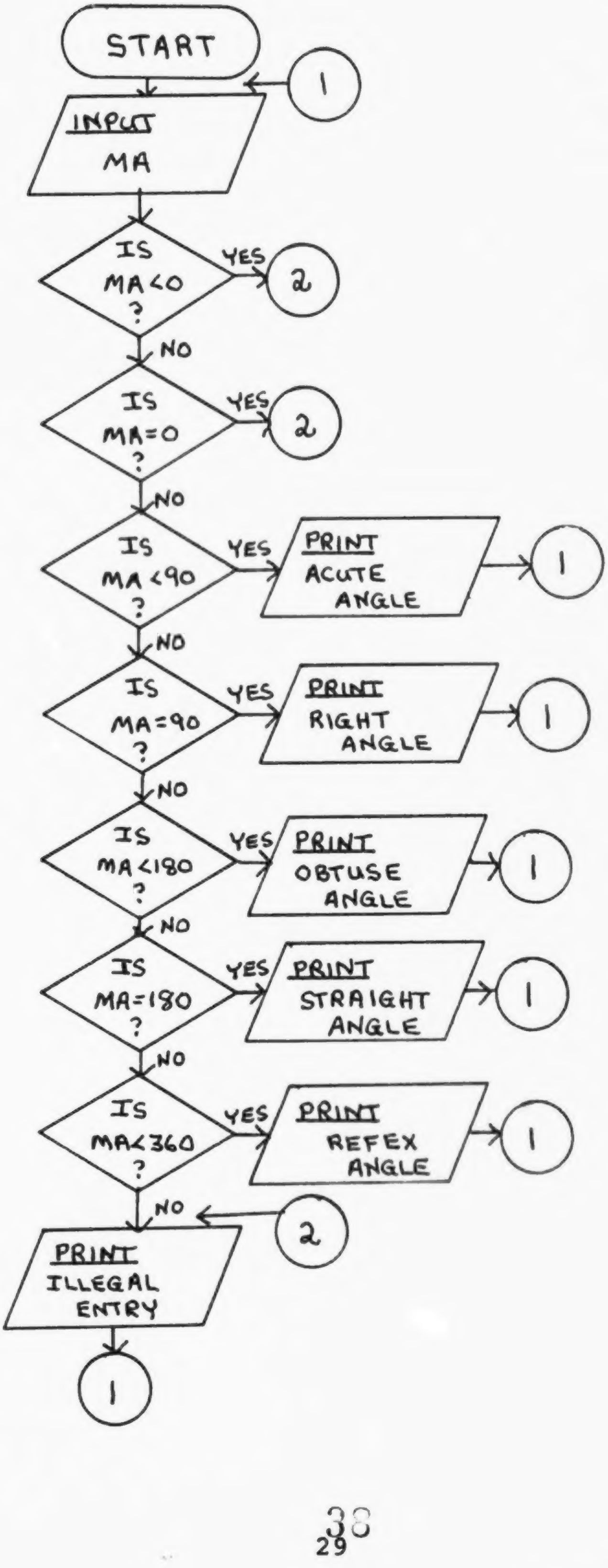
6. Ask the students to develop the plan at their seats. Circulate and help those who are having difficulty. Ask one or more students to write their plans on the chalkboard for class discussion.

Plan

```
 1)  Enter the measure of an angle, MA.
 2)  Check: Is MA < 0?
       a) If yes, go to 9.
       b) If no, continue.
 3)  Is MA = 0?
       a) If yes, go to 9.
       b) If no, continue.
 4)  Is MA < 90?
       a) If yes, print THE ANGLE IS AN ACUTE ANGLE.
          Go to 1.
       b) If no, continue.
 5)  Is MA = 90?
       a) If yes, print THE ANGLE IS A RIGHT ANGLE.
          Go to 1.
       b) If no, continue.
 6)  Is MA < 180?
       a) If yes, print THE ANGLE IS AN OBTUSE ANGLE. Go to 1.
       b) If no, continue.
 7)  Is MA = 180?
       a) If yes, print THE ANGLE IS A STRAIGHT ANGLE. Go to 1.
       b) If no, continue.
 8)  Is MA < 360?
       a) If yes, print THE ANGLE IS A REFLEX ANGLE. Go to 1.
       b) If no, continue.
 9)  Print ILLEGAL ENTRY.
10)  Go to 1.
```

7. Ask the students to flowchart the plan as follows.

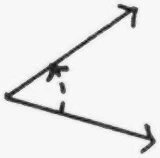
Flowchart


START
1
INPUT
MA
IS
MA<0
?
YES
2
NO
IS
MA=0
?
YES
2
NO
IS
MA<90
?
YES
PRINT
ACUTE
ANGLE
1
NO
IS
MA=90
?
YES
PRINT
RIGHT
ANGLE
1
NO
IS
MA<180
?
YES
PRINT
OBTUSE
ANGLE
1
NO
IS
MA=180
?
YES
PRINT
STRAIGHT
ANGLE
1
NO
IS
MA<360
?
YES
PRINT
REFEX
ANGLE
1
NO
2
PRINT
ILLEGAL
ENTRY
1

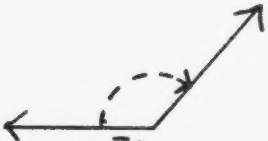
APPLICATIONS:

1. Estimate the measure of each of the following angles.

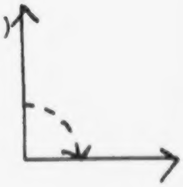
a)



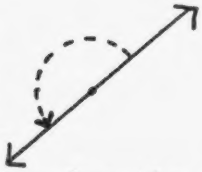
b)



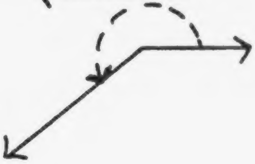
c)



d)



e)



2. Determine the classifications of the angles in problem 1.

3. Determine the classification of an angle with each of the following measures.
   a) 125 b) 30 c) 90 d) 270 e) 83 f) 180 g) 315
   h) 159 i) 89 j) 185

SUMMARY:

How do we plan the computer solution for determining the classification of an angle?

HOMEWORK;

1. Sketch an angle with each of the classifications acute, right, obtuse, straight and reflex.

2. Determine the classification of an angle with each of the following measures.
   a) 90 b) 8 c) 109 d) 190 e) 180

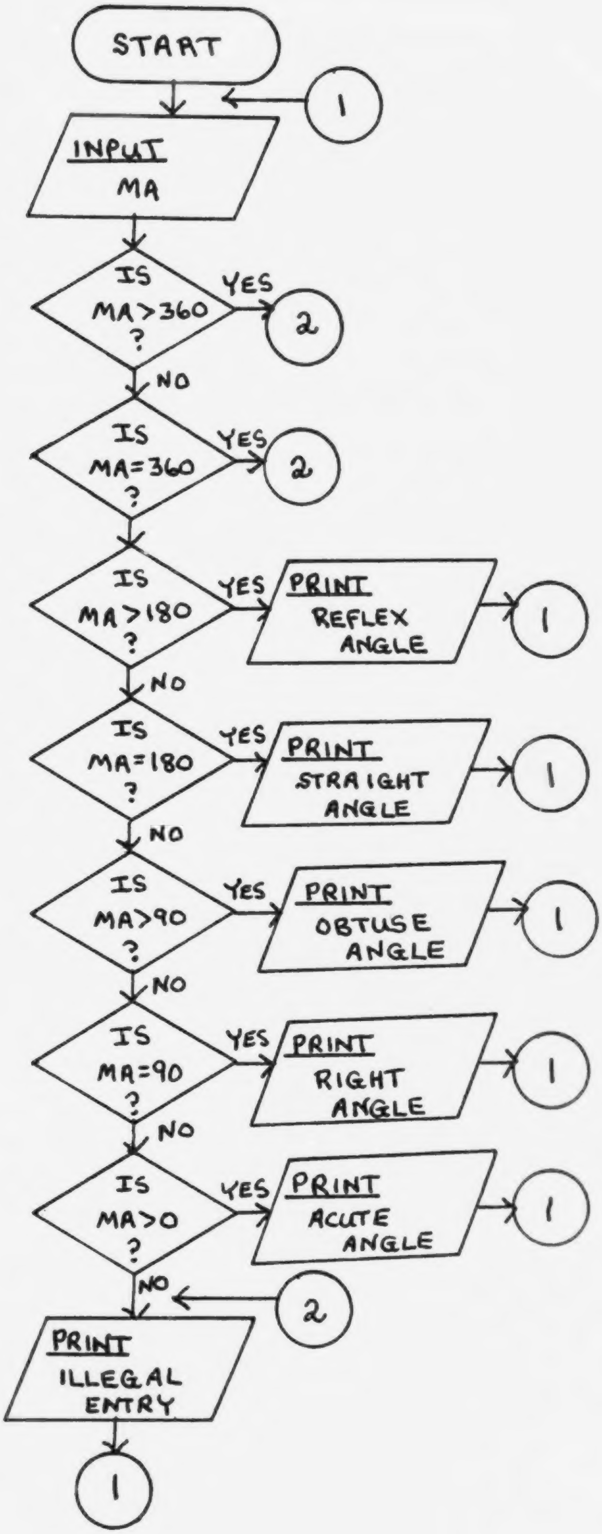
3. (For Extra Credit) Develop a flowchart for the classification of angles by determining first if an angle is a reflex angle.

HOMEWORK SOLUTIONS:

1. See Applications example 1.

2. a) right angle b) acute angle c) obtuse angle
   d) reflex angle e) straight angle

3. Flowchart


START
1
INPUT
MA
IS
MA>360
?
YES
2
NO
IS
MA=360
?
YES
2
IS
MA>180
?
YES
PRINT
REFLEX
ANGLE
1
NO
IS
MA=180
?
YES
PRINT
STRAIGHT
ANGLE
1
NO
IS
MA>90
?
YES
PRINT
OBTUSE
ANGLE
1
NO
IS
MA=90
?
YES
PRINT
RIGHT
ANGLE
1
NO
IS
MA>0
?
YES
PRINT
ACUTE
ANGLE
1
NO
2
PRINT
ILLEGAL
ENTRY
1

Lesson: 7L

AIM: How do we write a program to classify angles?

PERFORMANCE OBJECTIVES: The student will be able to...

1. design a planned output;
2. write a program using the flowchart and planned output;
3. test and debug the program.

VOCABULARY: planned output, prompt, debug, angle classification

MOTIVATION:

Use the flowchart developed in the previous lesson to determine the classification of each angle whose measure is listed below.

a) 129 b) 84 c) 269 d) 90 e) 270 f) 180 g) 45 h) 150

DEVELOPMENT:

1. Review the flowchart and the definition of each classification for an angle.
2. Ask, "What else must we plan prior to writing a program?"
3. Elicit a planned output as follows. Allow the students to develop their own planned outputs and remind them to plan for illegal input of data.

Planned Output

```
CLASSIFYING ANGLES
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360.
? 90
(Skip 1 line.)
AN ANGLE WHICH MEASURES 90 DEGREES IS A RIGHT ANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360.
?-65
(Skip 1 line.)
ILLEGAL ENTRY!
```

```
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360.
? 450
(Skip 1 line.)
ILLEGAL ENTRY!
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360.
? 360
(Skip 1 line.)
ILLEGAL ENTRY!
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360.
? 150
(Skip 1 line.)
AN ANGLE WHICH MEASURES 150 DEGREES IS AN OBTUSE ANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360.
?
```

4. Elicit the program which follows. In this program, we will introduce multiple statements on a line. Therefore, it is recommended that this program be developed with the students at the chalkboard.

<u>Program</u>

```
10     PRINT "CLASSIFYING ANGLES"
20     PRINT:PRINT
30     PRINT "ENTER THE MEASURE OF AN ANGLE BETWEEN 0 AND 360."
40     INPUT MA:PRINT
50     IF MA < 0 THEN GOTO 120
60     IF MA = 0 THEN GOTO 120
70     IF MA < 90 THEN PRINT "AN ANGLE WHICH MEASURES ";MA;"
DEGREES IS AN ACUTE ANGLE.":GOTO 20
80     IF MA = 90 THEN PRINT "AN ANGLE WHICH MEASURES ";MA;"
DEGREES IS A RIGHT ANGLE.":GOTO 20
90     IF MA < 180 THE PRINT " AN ANGLE WHICH MEASURES ";MA;"
DEGREES IS AN OBTUSE ANGLE.":GOTO 20
100    IF MA = 180 THEN PRINT " AN ANGLE WHICH MEASURES ";MA;"
DEGREES IS A STRAIGHT ANGLE.":GOTO 20
```

```
110   IF MA < 360 THEN PRINT " AN ANGLE WHICH MEASURES ";MA;"
DEGREES IS A REFLEX ANGLE.":GOTO 20
120   PRINT "ILLEGAL ENTRY!":GOTO 20
130   END
```

5. Ask the students to enter, run, and test their programs using the angle measures in the Application example.

APPLICATION:

Use the program written in today's lesson in order to classify, where possible, the angle measures listed below.

a) 67 b) 0 c) 60 d) 90 e) 180 f) 135
g) 320 h) 360 i) 450

SUMMARY:

1. What procedure do we use to code multiple statements on a line? What is the advantage or disadvantage?
2. Why are the following statements not needed in the program?

```
IF MA = 360 THEN GOTO 120
IF MA > 360 THEN GOTO 120
```

HOMEWORK:

1. What is the sum of the measures of two-thirds of a straight angle and one-half of a right angle?
2. Complete each statement with the word **acute, obtuse** or **right.**
   - a) If an angle is acute, then its complement is _____.
   - b) If an angle is acute, then its supplement is _____.
   - c) If an angle is right, then its supplement is _____.
   - d) If two angles of a triangle are complementary, then the third angle is a(n) _____ angle.
   - e) The type of angle suggested by the hands of a clock when it shows 7 o'clock is a(n) _____ angle.

f) The type of angle suggested by the hands of a clock when it shows 10 minutes after 1 is a(n) _____ angle.

g) If three angles of a quadrilateral measure 83°, 77°, and 110°, the third angle is a(n) _____ angle.

3. Write a program to classify angles which uses the flowchart developed in answer to problem #3 in the previous lesson's homework and the following planned output.

   (Note to teacher: For those students who did not succeed in developing the flowchart for extra credit, ask a student who did, to write his/her flowchart on the chalkboard or present his/her flowchart on a rexograph stencil along with the planned output.)

   Planned Output

```
CLASSIFYING ANGLES
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE.
? 90
RIGHT ANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE.
? 360
ILLEGAL ENTRY!
(Skip 2 lines.)
ENTER THE MEASURE OF AN ANGLE.
?
```

HOMEWORK SOLUTIONS:

1. 165

2. a) acute
   b) obtuse
   c) right
   d) right
   e) obtuse
   f) acute
   g) right

3. Program

```
10     PRINT "CLASSIFYING ANGLES"
20     PRINT:PRINT
30     PRINT "ENTER THE MEASURE OF AN ANGLE."
40     INPUT MA
50     IF MA > 360 THEN GOTO 120
60     IF MA = 360 THEN GOTO 120
70     IF MA > 180 THEN PRINT "REFLEX ANGLE.":GOTO 20
80     IF MA = 180 THEN PRINT "STRAIGHT ANGLE.":GOTO 20
90     IF MA > 90 THEN PRINT "OBTUSE ANGLE.":GOTO 20
100    IF MA = 90 THEN PRINT "RIGHT ANGLE.":GOTO 20
110    IF MA > 0 THEN PRINT "ACUTE ANGLE.":GOTO 20
120    PRINT "ILLEGAL ENTRY!":GOTO 20
130    END
```

Lesson: 8

AIM: How do we classify triangles according to the measures of their angles?

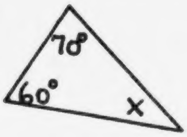
PERFORMANCE OBJECTIVES: The student will be able to...

1. determine the measure of the third angle of a triangle when given the measures of two angles;
2. classify triangles according to the measures of their angles.

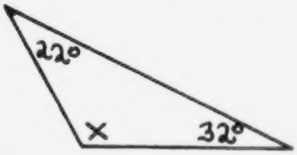
VOCABULARY: sum, acute triangle, right triangle, obtuse triangle

MOTIVATION: Determine the unknown angle measure in each triangle.

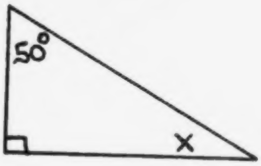
a)



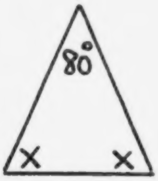
c)



b)



d)



DEVELOPMENT:

1. Elicit responses to the motivation examples. Review the theorem, "The sum of the measurements of the angles of a triangle is 180 degrees." (See Part I, Lesson 33.)

2a. Ask, "How can we determine the measure of the unknown angle?"

2b. Elicit that in any triangle ABC the following algorithm can be used to determine the measure of the third angle:
A = 180-(B+C) or B = 180-(A+C) or C = 180-(A+B)

3. Ask the students to draw a triangle with two right angles.

   a) Elicit that a triangle can contain one and only one right angle.

   - b) Define a right triangle as a triangle with one right angle.

4. Ask, "How many obtuse angles can a triangle contain?"
   - a) Elicit that a triangle can contain one and only one obtuse angle.
   - b) Define an obtuse triangle as a triangle with one obtuse angle.

5. Ask, "How many acute angles can a triangle contain?"
   - a) Elicit that a triangle can contain two or three acute angles.
   - b) Define an acute triangle as a triangle with three acute angles.

6. Ask the students to complete the Applications examples.

APPLICATIONS:

1. Determine the classification of a triangle which contains each the following angle measurements.
   - a) 60°, 60°, 60°
   - c) 10°, 150°, 20°
   - b) 45°, 45°, 90°
   - d) 25°, 85°, 70°

2. Determine the classification of a triangle which contains each the following pairs of angle measurements.
   - a) 30°, 60° b) 18°, 20° c) 50°, 75° d) 45°, 90°

SUMMARY:

1. Can you classify a triangle if you are told that it has two acute angles? Why or why not?

2. Given the measures of two angles of a triangle, how can the measure of the third angle be determined?

3. Is there a "reflex triangle"? Explain.

HOMEWORK:

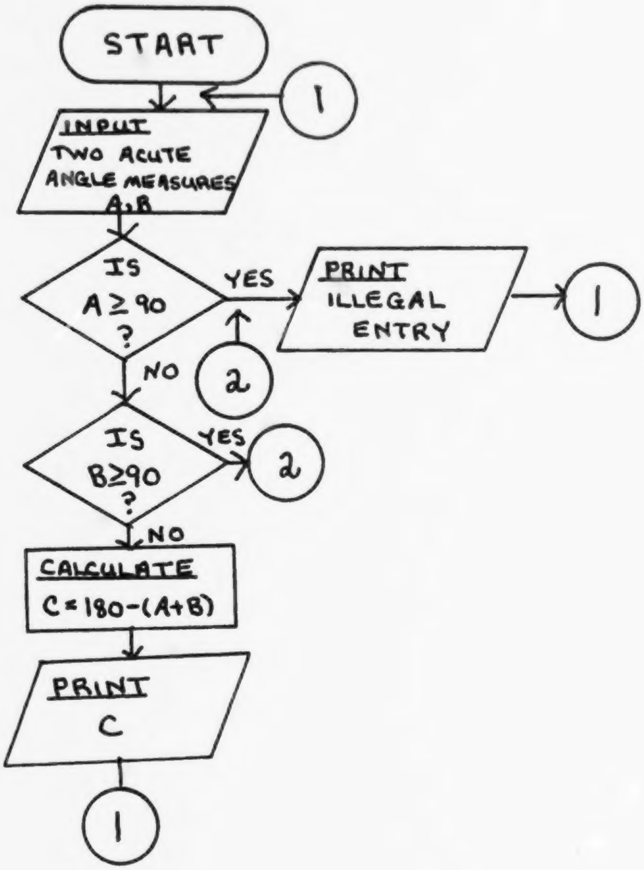
1. Determine the classification of a triangle which contains each the following pairs of acute angle measurements.
   - a) 32°, 78°　b) 19°, 34°　c) 56°, 34°
   - d) 89°, 89°　e) 45°, 45°
2. Develop a flowchart which can be used as a plan for the solution of the following problem.

> Problem:
>
> Have the computer determine and display the measure of the third angle of a triangle, given the measures of two acute angles of the triangle.

HOMEWORK SOLUTIONS:

1.
   - a) acute triangle
   - b) obtuse triangle
   - c) right triangle
   - d) acute triangle
   - e) right triangle

2. Flowchart


START
1
INPUT
TWO ACUTE
ANGLE MEASURES
A,B
IS
A ≥ 90
?
YES
PRINT
ILLEGAL
ENTRY
1
NO
2
IS
B≥90
?
YES
2
NO
CALCULATE
C=180-(A+B)
PRINT
C
1

Lesson: 9L

AIM: How do we write a program to have the computer determine the classification of a triangle?

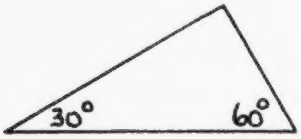
PERFORMANCE OBJECTIVES: The student will be able to...

1. incorporate the algorithm C = 180-(A+B) into a plan to determine the classification of a triangle, given the measures of two acute angles;
2. flowchart a computer solution;
3. write a complete computer solution.

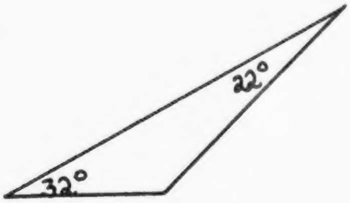
VOCABULARY: acute triangle, obtuse triangle, right triangle

MOTIVATION: Classify each of the following triangles as acute, obtuse or right.

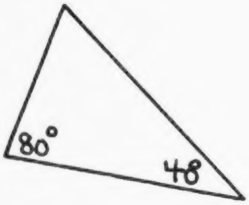
a)



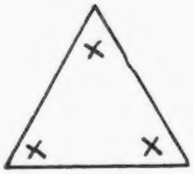
b)



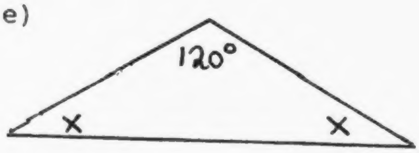
c)



d)



e)



DEVELOPMENT:

1. Elicit the solutions to the motivation examples. Review the answer to homework question #2, Lesson 8. (A flowchart for determining the measure of the third angle of a triangle when given the measures of two acute angles.)

2. Ask, "How can we incorporate the algorithm for finding the measure of the third angle into a plan for determining the classification of a triangle?"

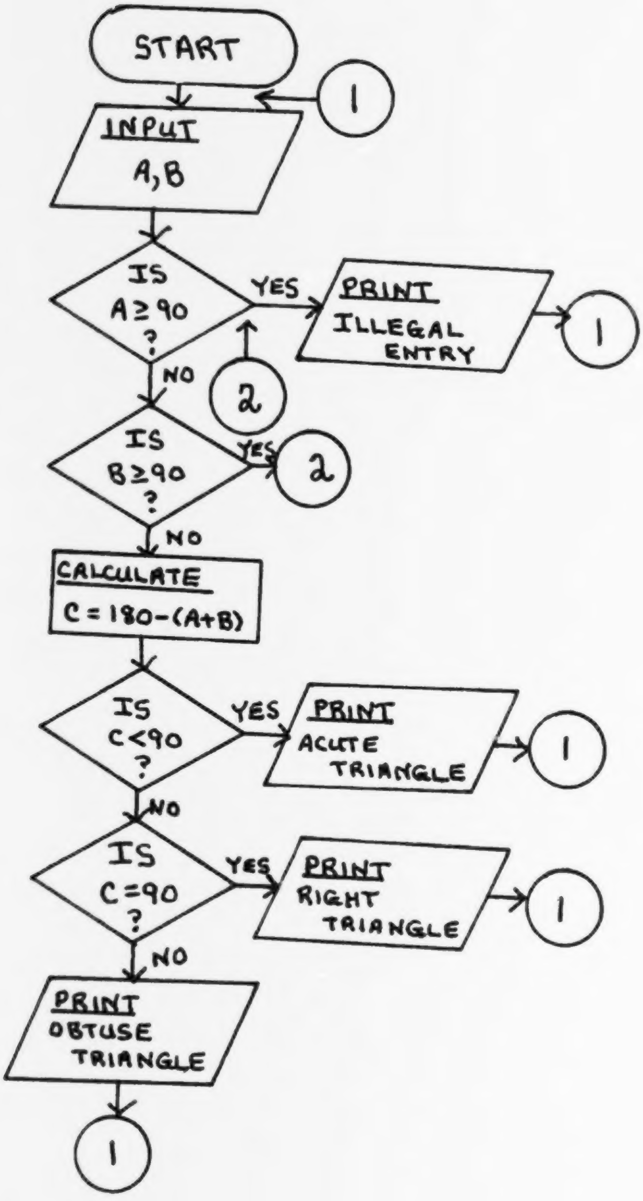
3. Elicit the plan which follows.

   <u>Plan</u>

   ```
   1)  Enter the measure of two acute angles: A,B.
   2)  Check to see if the data is "correct."
         Ask, "What does 'correct' mean here?"
   3)  Calculate the measure of the third angle:
         $C=180-(A+B)$.
   4)  Check: Is $C<90$?
         a)  If yes, then print ACUTE TRIANGLE.  Go to 1.
         b)  If no, continue.
   5)  Check: Is $C=90$?
         a)  If yes, then print RIGHT TRIANGLE.  Go to 1.
         b)  If no, continue.
   6)  Print OBTUSE TRIANGLE.
   7)  Go to 1.
   ```

4. Ask the students to flowchart the plan as follows. (The students who completed homework question #2/Lesson 8 correctly, can extend their original flowcharts.)

Flowchart


START
INPUT
A,B
IS
A≥90
?
YES
PRINT
ILLEGAL
ENTRY
1
NO
2
IS
B≥90
?
YES
2
NO
CALCULATE
C = 180-(A+B)
IS
C<90
?
YES
PRINT
ACUTE
TRIANGLE
1
NO
IS
C=90
?
YES
PRINT
RIGHT
TRIANGLE
1
NO
PRINT
OBTUSE
TRIANGLE
1

5. Ask the students to design a planned output as follows.

Planned Output

```
CLASSIFYING TRIANGLES BY ANGLES
(Skip 2 lines.)
ENTER THE MEASURES OF TWO ACUTE ANGLES.
? 30,40
THIS IS AN OBTUSE TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURES OF TWO ACUTE ANGLES.
? 75,125
INCORRECT ENTRY!
(Skip 2 lines.)
ENTER THE MEASURES OF TWO ACUTE ANGLES.
? 45,45
THIS IS A RIGHT TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURES OF TWO ACUTE ANGLES.
?
```

6. ask the students to code, enter and run the program. Remind them to include all parts of the planned output.

Program

```
10    PRINT "CLASSIFYING TRIANGLES BY ANGLES"
20    PRINT:PRINT
30    PRINT "ENTER THE MEASURES OF TWO ACUTE ANGLES."
40    INPUT A,B
50    IF A>=90 THEN PRINT "INCORRECT ENTRY!":GOTO 20
60    IF B>=90 THEN PRINT "INCORRECT ENTRY!":GOTO 20
70    C=180-(A+B)
80    IF C< 90 THEN PRINT "THIS IS AN ACUTE TRIANGLE.":GOTO 20
90    IF C=90 THEN PRINT "THIS IS A RIGHT TRIANGLE.":GOTO 20
100   PRINT "THIS IS AN OBTUSE TRIANGLE.":GOTO 20
110   END
```

7. Ask the students to complete the Applications examples.

APPLICATIONS:

1. Using your program and the following pairs of acute angle measurements, classify each triangle as acute, obtuse or right.
   a) 35°, 67°   b) 48°, 79°   c) 72°, 18°   d) 5°, 18°   e) 55°, 55°
2. Listed below are measurements for the base angles of isosceles triangles. For each, use your program to determine the classification, according to the measurements of the angles, of the isosceles triangle.
   a) 45°   b) 75°   c) 33°   d) 60°   e) 17°
3. Given the measurement of the vertex angle of an isosceles triangle in each, determine the triangle's classification according to the measurements of its angles.
   a) 90°   b) 150°   c) 60°   d) 70°   e) 118°

SUMMARY:

1. Why did we only allow the user to enter the measures of two acute angles in our program for classifying triangles?
2. a) Examine your responses to question #2 in the Applications. Knowing the measure of the base angles and without computing the measure of the vertex angle, under what circumstances will the triangle be a right triangle? ... an acute triangle? ... an obtuse triangle?

   b) By inspection only, classify each isosceles triangle, according to the measurements of its angles, if the measurements of its base angles are...
   - i) 71°;
   - ii) 6°;
   - iii) 45°.

HOMEWORK:

Complete the instructions for each of the following problems.

a) Write an analysis which includes an algorithm.
b) Write a plan.
c) Develop a flowchart.
d) Design a planned output.
e) Code a program to solve the problem.

Problems:

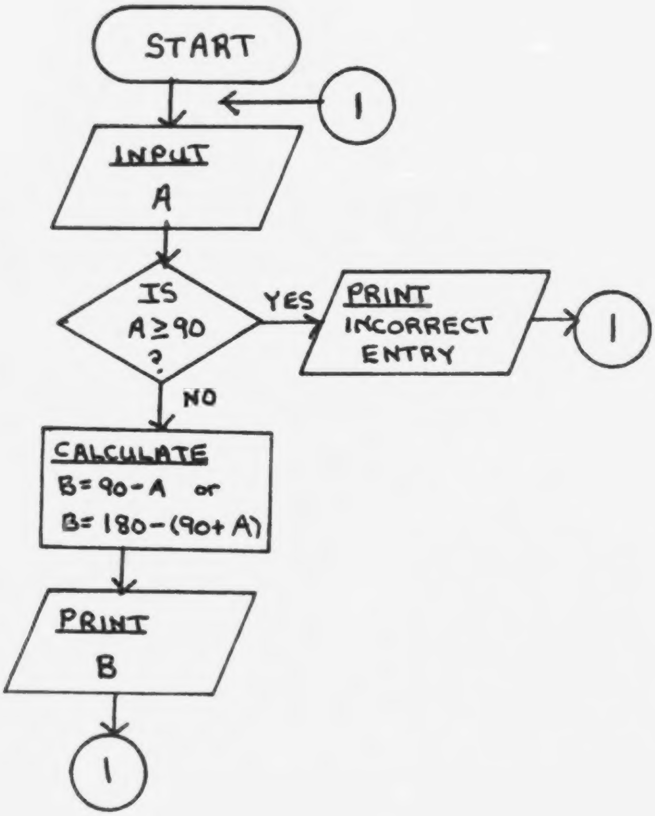
1. Enter the measure of one acute angle of a right triangle and have the computer find and display the measure of the other acute angle. (Assume that the user will enter a positive number.)
2. Enter the measure of the vertex angle of an isosceles triangle and have the computer find and display the measure of each base angle. (Assume that the user will enter a positive number.)
3. Enter the measure of the vertex angle of an isosceles triangle and have the computer classify the triangle according to the measures of its angles. (Assume that the user will enter a positive number.)
4. Enter the measure of one base angle of an isosceles triangle. Without having the computer calculate the measure of the vertex angle, have it classify the triangle according to the measures of its angles. (Assume that the user will enter a positive number.)

(Note: The students should be allowed more than one day to complete the computer solution to a problem. Also, it is suggested that these be completed by the students as independent projects.)

HOMEWORK SOLUTIONS:

(A suggested flowchart, planned output and program for each problem is provided. The required analyses and plans can be gleaned from these.)

1c) Flowchart


START
1
INPUT
A
IS
A ≥ 90
?
YES
PRINT
INCORRECT
ENTRY
1
NO
CALCULATE
B = 90 - A or
B = 180 - (90 + A)
PRINT
B
1

1d) Planned Output

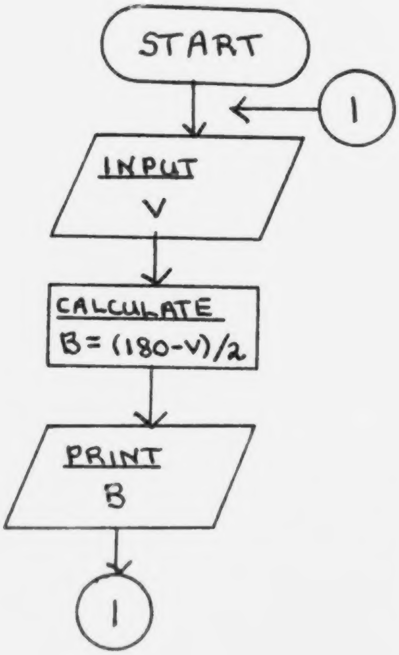
```
FINDING THE MEASURE OF AN ACUTE ANGLE
*** OF A RIGHT TRIANGLE ***
(Skip 2 lines.)
ENTER THE MEASURE OF AN ACUTE ANGLE.
? 30
THE MEASURE OF THE OTHER ACUTE ANGLE
IS 60 DEGREES.
(Skip 2 lines.)
ENTER THE MEASURE OF AN ACUTE ANGLE.
? 105
INCORRECT ENTRY!
(Skip 2 lines.)
ENTER THE MEASURE OF AN ACUTE ANGLE.
?
```

1e) Program

```
10  PRINT "FINDING THE MEASURE OF AN ACUTE ANGLE"
20  PRINT "*** OF A RIGHT TRIANGLE ***"
30  PRINT:PRINT
40  PRINT "ENTER THE MEASURE OF AN ACUTE ANGLE."
50  INPUT A
60  IF A>=90 THEN PRINT "INCORRECT ENTRY!":GOTO 30
70  B=90-A
80  PRINT "THE MEASURE OF THE OTHER ACUTE ANGLE"
90  PRINT "IS"; B; "DEGREES."
100 GOTO 30
110 END
```

2. See the next page.

2c) Flowchart


START
1
INPUT
V
CALCULATE
B = (180-V)/2
PRINT
B
1

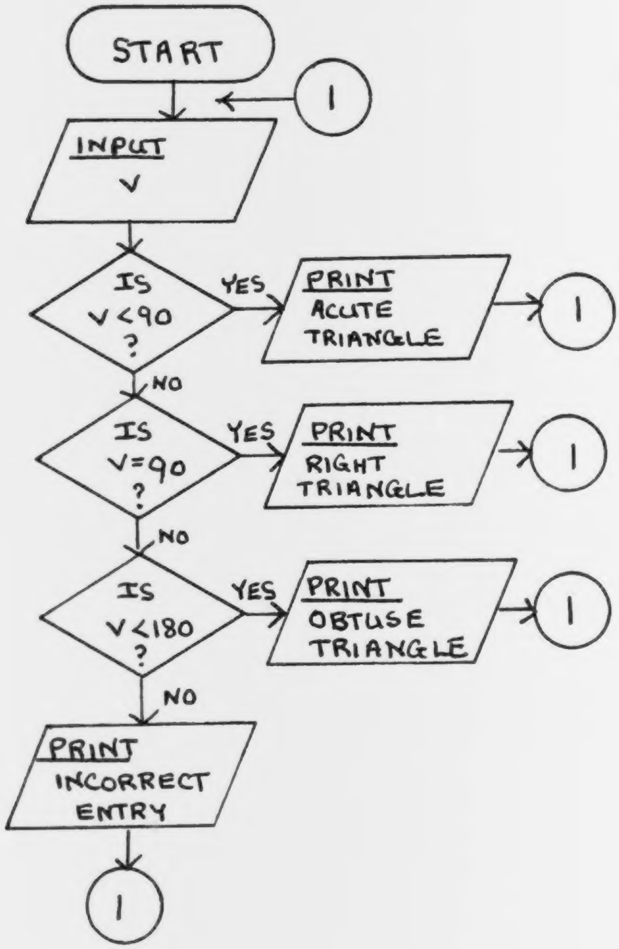
2d) Planned Output

```
FINDING THE MEASURE OF THE BASE ANGLES
*** OF AN ISOSCELES TRIANGLE ***
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
? 80
EACH BASE ANGLE MEASURES 50 DEGREES.
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
?
```

2e) Program

```
10   PRINT "FINDING THE MEASURE OF THE BASE ANGLES"
20   PRINT "*** OF AN ISOSCELES TRIANGLE ***"
30   PRINT:PRINT
40   PRINT "ENTER THE MEASURE OF THE VERTEX ANGLE."
50   INPUT V
60   B=(180-V)/2
70   PRINT "EACH BASE ANGLE MEASURES"; B; "DEGREES."
80   GOTO 30
90   END
```

3c) Flowchart


START
1
INPUT
V
IS
V<90
?
YES
PRINT
ACUTE
TRIANGLE
1
NO
IS
V=90
?
YES
PRINT
RIGHT
TRIANGLE
1
NO
IS
V<180
?
YES
PRINT
OBTUSE
TRIANGLE
1
NO
PRINT
INCORRECT
ENTRY
1

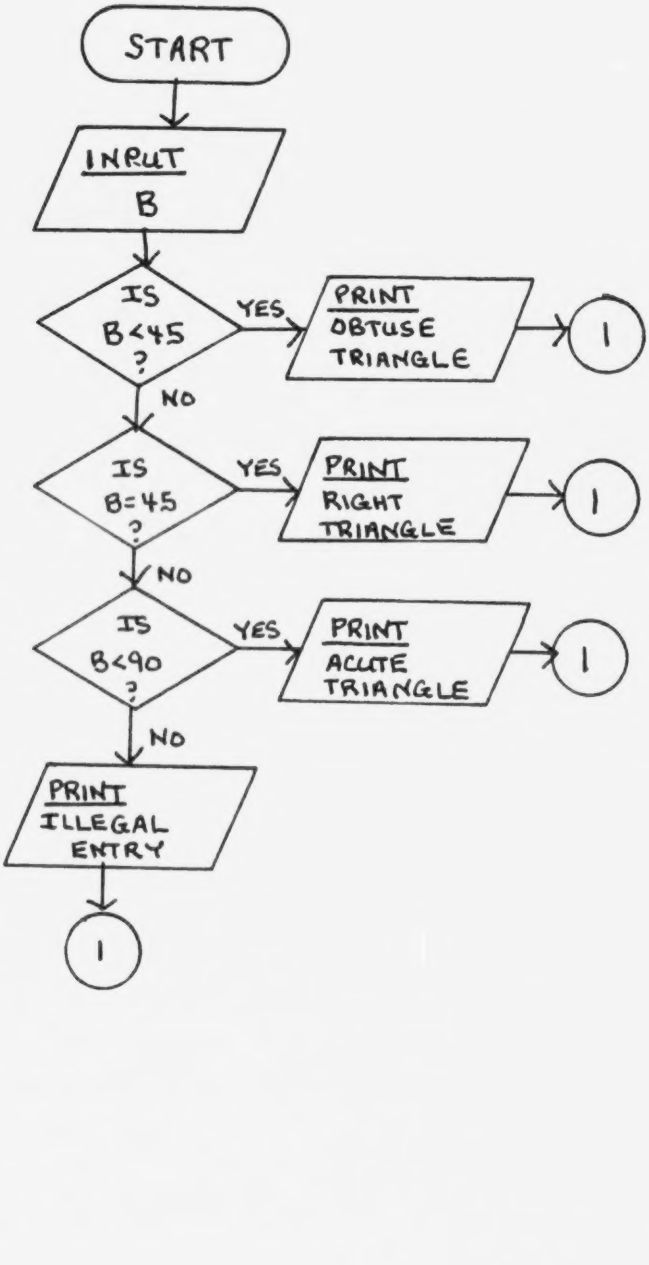
3d) Planned Output

```
CLASSIFYING AN ISOSCELES TRIANGLE FROM THE MEASURE
*** OF ITS VERTEX ANGLE ***
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
? 190
INCORRECT ENTRY!
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
? 80
THIS IS AN ACUTE TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
? 90
THIS IS A RIGHT TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
? 120
THIS IS AN OBTUSE TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF THE VERTEX ANGLE.
?
```

3e) Program

```
10  PRINT "CLASSIFYING AN ISOSCELES TRIANGLE FROM THE MEASURE"
20  PRINT "*** OF ITS VERTEX ANGLE ***"
30  PRINT:PRINT
40  PRINT "ENTER THE MEASURE OF THE VERTEX ANGLE."
50  INPUT V
60  IF V< 90 THEN PRINT "THIS IS AN ACUTE TRIANGLE.":GOTO 30
70  IF V=90 THEN PRINT "THIS IS A RIGHT TRIANGLE.":GOTO 30
80  IF V<180 THEN PRINT "THIS IS AN OBTUSE TRIANGLE.":GOTO 30
90  PRINT "INCORRECT ENTRY!":GOTO 30
100 END
```

4c) Flowchart


START
INPUT
B
IS
B<45
?
YES
PRINT
OBTUSE
TRIANGLE
1
NO
IS
B=45
?
YES
PRINT
RIGHT
TRIANGLE
1
NO
IS
B<90
?
YES
PRINT
ACUTE
TRIANGLE
1
NO
PRINT
ILLEGAL
ENTRY
1

4d) Planned Output

```
CLASSIFYING AN ISOSCELES TRIANGLE FROM THE MEASURES
*** OF ITS BASE ANGLES ***
(Skip 2 lines.)
ENTER THE MEASURE OF ONE BASE ANGLE.
? 95
INCORRECT ENTRY!
(Skip 2 lines.)
ENTER THE MEASURE OF ONE BASE ANGLE.
? 10
THIS IS AN OBTUSE TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF ONE BASE ANGLE.
? 45
THIS IS A RIGHT TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF ONE BASE ANGLE.
? 65
THIS IS AN ACUTE TRIANGLE.
(Skip 2 lines.)
ENTER THE MEASURE OF ONE BASE ANGLE.
?
```

4e) Program

```
10  PRINT "CLASSIFYING AN ISOSCELES TRIANGLE FROM THE MEASURES"
20  PRINT "*** OF ITS BASE ANGLES ***"
30  PRINT:PRINT
40  PRINT "ENTER THE MEASURE OF ONE BASE ANGLE."
50  INPUT B
60  IF B<45 THEN PRINT "THIS IS AN OBTUSE TRIANGLE.":GOTO 30
70  IF B=45 THEN PRINT "THIS IS A RIGHT TRIANGLE.":GOTO 30
80  IF B<90 THEN PRINT "THIS IS AN ACUTE TRIANGLE.":GOTO 30
90  PRINT "INCORRECT ENTRY!":GOTO 30
100 END
```

Lesson: 10

AIM: How do Fahrenheit and Celsius temperatures compare?

PERFORMANCE OBJECTIVES: The student will be able to...

1. identify the freezing point of water, the boiling point of water and other Fahrenheit and Celsius temperature readings;
2. convert from Celsius measure to Fahrenheit measure using the formula $F = 9C/5+32$;
3. convert from Fahrenheit measure to Celsius measure using the formula $C = 5/9(F-32)$.

VOCABULARY: standard unit, Fahrenheit, Celsius, conversion

MOTIVATION:

Jimmy's mother heard on the radio that today's temperature is 32° Celsius and insisted that he wear his heavy winter coat. Why did Jimmy protest and say that he did not need it?

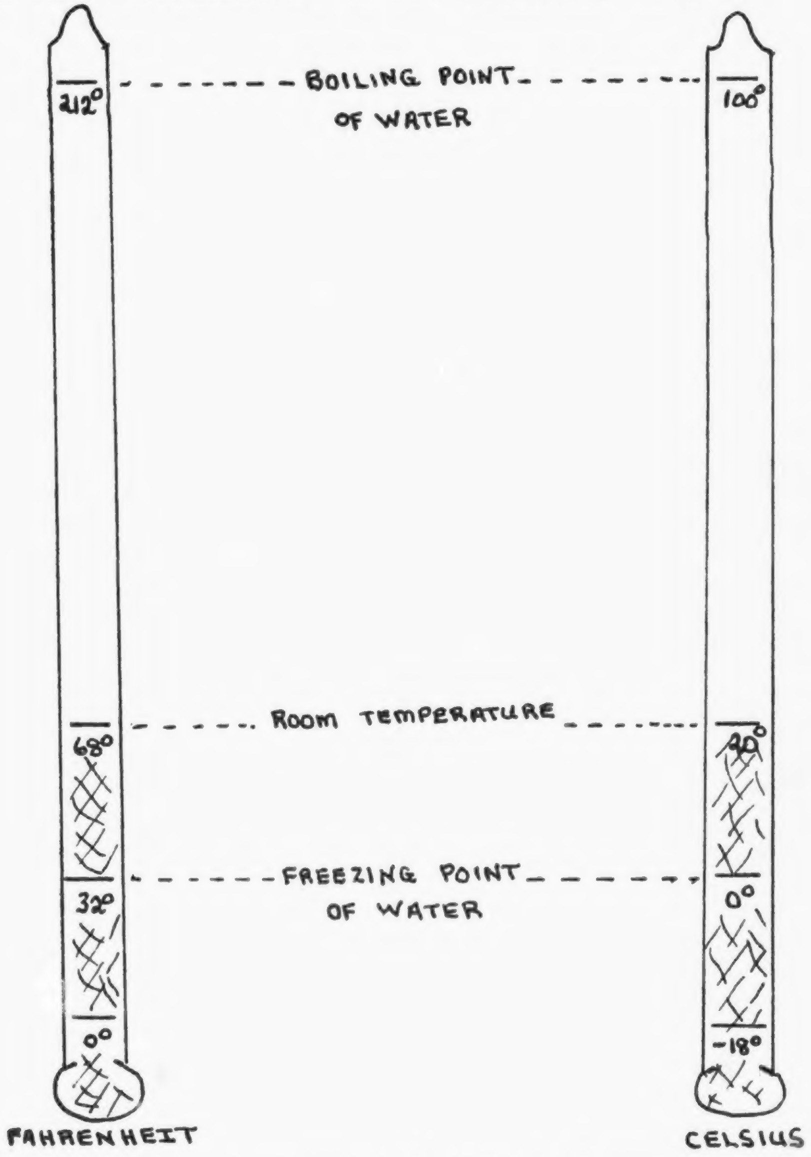
DEVELOPMENT:

1. Discuss the motivation with the students. Briefly review the history and originators of the two standard units used when measuring temperature...

   Fahrenheit, Gabriel Daniel, a German physicist who in 1714 constructed the first thermometer using mercury and devised the temperature scale known by his name.

   Celsius, Anders, a Swedish astronomer who in 1742 devised a thermometer whose scale is based on 100 (also called Centigrade scale).

2. Demonstrate the differences between the thermometers with either real thermometers or the following diagram.


BOILING POINT OF WATER
212°
100°
ROOM TEMPERATURE
68°
20°
FREEZING POINT OF WATER
32°
0°
0°
-18°
FAHRENHEIT
CELSIUS

3. Ask the students to compare the two scales as follows.
   a) Ask, "Which scale has more markings?"
   b) Ask, "How many degrees are there between the boiling and freezing points on each thermometer?"
   c) Ask, "What is the ratio of Fahrenheit degrees to Celsius degrees?" (F:C = 180:100 or F/C = 9/5)
   d) Ask, "How can we convert a given number of degrees from one unit to the other?" (Use F = 9/5xC and C = 5/9xF.)

4. Ask the students to complete Applications examples 1 and 2.
5. Elicit the conversion formulae as follows.
   a) Ask, "At the freezing point, what is the difference in the reading between the Fahrenheit and Celsius scales?"
   b) Ask, "How can we adjust the formulae for converting from degrees in one temperature scale to the other?"(+32 or -32) Hence $F = 9/5C+32$ or $C=5/9(F-32)$.
6. Ask the students to complete Applications examples 3-5.

APPLICATIONS:

1. From 6:00 A.M. until 3:00 P.M. during a summer day the temperature went up 15°C. How many degrees Fahrenheit did the temperature rise?
2. From 3:00 P.M. until 10:00 P.M. the temperature dropped 18°F. How many degrees Celsius did the temperature drop?
3. Convert each to the equivalent Fahrenheit temperature reading.
   a) 0°C b) 50°C c) 10°C d) 32°C
4. Convert each to the equivalent Celsius temperature reading.
   a) 32°F b) 50°F c) 212°F d) 95°F
5. Using the appropriate conversion formula, convert each temperature reading to its equivalent on the other temperature scale.
   a) 25°C b) 68°F c) 30°C d) 41°F

SUMMARY:

1. Why do we derive(get) 9/5 and the reciprocal 5/9 in the conversion forumlae?
2. Why do we derive(get) +32 and -32 in the conversion formulae?
3. How can we remember which is the appropriate formula to use?

4. What is the normal body temperature reading on a Fahrenheit thermometer? ... Celsius thermometer?

HOMEWORK:

1. Using the appropriate conversion formula, convert each temperature reading to its equivalent on the other temperature scale.
   a) 5°C b) 59°F c) 140°F d) 37°C e) -10°C

2. If the temperature dropped 35°C, how much did it drop in Fahrenheit units?

3. If the temperature at 12:00 noon is 77°F and at 4:00 P.M. is 86°F, how many degrees Celsius did the temperature rise? Also, what was the Celsius temperature reading at 4:00 P.M.?

HOMEWORK SOLUTIONS:

1. a) 41°F b) 15°C c) 60°C d) 98.6°F e) 14°F
2. 63°F
3. 5°C, 30°C

Lesson: 11

AIM: How do we code a program to convert Fahrenheit and Celsius temperatures?

PERFORMANCE OBJECTIVES: The student will be able to...

1. incorporate the forumlae for temperature conversion into a plan for a computer solution;
2. flowchart a computer solution;
3. design a planned output;
4. write a program.

VOCABULARY: Fahrenheit temperature, Celsius temperature, conversion, prompt

MOTIVATION:

Compare the two given temperatures and determine whether the first is the colder temperature, warmer temperature or the same temperature as the second.

1) 45°C, 112°F
2) 8°C, 50°F
3) 37°C, 98.6°F
4) 27°C, 49°F

DEVELOPMENT:

1. Elicit solutions to the motivation problem. In each exercise, ask which temperature reading would be easier to convert and review the appropriate formula for the conversion.
2. Present the following problem for computer solution.

   Problem: If a temperature is entered in Celsius or Fahrenheit degrees, have the computer convert it to the other.

3. Elicit the analysis as follows.

Analysis

a) Ask, "Which algorithms will we use to do the conversions? How can you determine which algotithm to use for each?"

b) Say to the students that it will be necessary for the user to communicate whether the temperature to be converted is measured in Celsius or Fahrenheit degrees. This can be accomplished by using a numerical code; e.g., 1 will represent converting Fahrenheit to Celsius and 2 will represent converting Celsius to Fahrenheit. A check of the number entered will indicate which formula to use in the conversion.

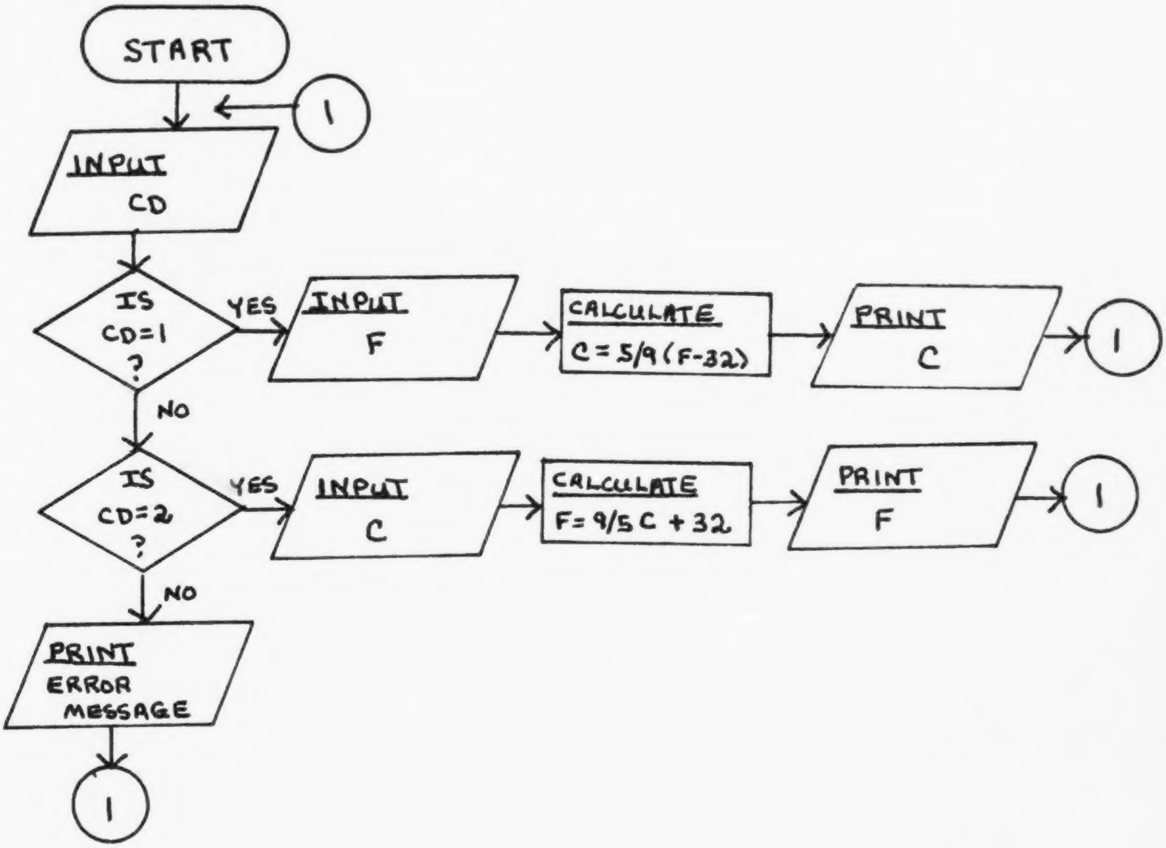
4. Elicit the plan which follows.

Plan

```
1) Print a message telling the user the numerical code.
2) Enter the code number: CD.
3) Check: Is CD=1?
    a)  If yes, go to 5.
    b)  If no, continue.
4) Check: Is CD=2?
    a)  If yes, go to 9.
    b)  If no, print an error message and go to 2.
5) Enter a Fahrenheit temperature: F.
6) Calculate: $C=\frac{5}{9}(F-32)$.
7) Print C.
8) Go to 1.
9) Enter a Celsius temperature: C.
10) Calculate: $F=\frac{9}{5}C+32$.
11) Print F.
12) Go to 1.
```

5. Ask the students to flowchart the plan as follows.

Flowchart


START
1
INPUT CD
IS CD=1 ?
YES
INPUT F
CALCULATE C=5/9(F-32)
PRINT C
1
NO
IS CD=2 ?
YES
INPUT C
CALCULATE F=9/5C+32
PRINT F
1
NO
PRINT ERROR MESSAGE
1

6. Ask the students to trace the flowchart using the appropriate temperatures from the motivation.
7. Ask the students to design a planned output as follows.

Planned Output

```
TEMPERATURE CONVERSION
(Skip 2 lines.)
ENTER 1 TO CONVERT FAHRENHEIT TO CELSIUS
AND 2 TO CONVERT CELSIUS TO FAHRENHEIT.
? 1
ENTER THE FAHRENHEIT TEMPERATURE? 50
(Skip 1 line.)
50 DEGRESS FAHRENHEIT = 10 DEGREES CELSIUS.
(Skip 2 lines.)
ENTER 1 TO CONVERT FAHRENHEIT TO CELSIUS
AND 2 TO CONVERT CELSIUS TO FAHRENHEIT.
? 3
```

```
INCORRECT ENTRY! PLEASE REENTER.
? 2
ENTER THE CELSIUS TEMPERATURE? 37
(Skip 1 line.)
37 DEGREES CELSIUS = 98.6 DEGREES FAHRENHEIT.
(Skip 2 lines.)
ENTER 1 TO CONVERT FAHRENHEIT TO CELSIUS
AND 2 TO CONVERT CELSIUS TO FAHRENHEIT.
?
```

8. Ask the students to code the program by referring to the flowchart and the planned output. Introduce the BASIC syntax which combines the prompt to the user and keyboard entry in one statement; e.g., INPUT "message"; variable.

Program

```
10 PRINT "TEMPERATURE CONVERSION"
20 PRINT:PRINT
30 PRINT "ENTER 1 TO CONVERT FAHRENHEIT TO CELSIUS"
40 PRINT "AND 2 TO CONVERT CELSIUS TO FAHRENHEIT."
50 INPUT CD
60 IF CD=1 THEN GOTO 90
70 If CD=2 THEN GOTO 140
80 PRINT "INCORRECT ENTRY! PLEASE REENTER.":GOTO 50
90 INPUT "ENTER THE FAHRENHEIT TEMPERATURE";F
100 C=5/9*(F-32)
110 PRINT
120 PRINT F; "DEGREES FAHRENHEIT = ";C; " DEGREES
CELSIUS."
130 GOTO 20
140 INPUT "ENTER THE CELSIUS TEMPERATURE";C
150 F=9/5*C+32
160 PRINT
170 PRINT C; "DEGREES CELSIUS = ";F;" DEGREES
FAHRENHEIT."
180 GOTO 20
190 END
```

APPLICATIONS:

1. Trace your program using the following temperatures.
   a) 0°C b) 212°F c) 20°C d) 83°C e) 45°F
2. Convert . . .
   - a) 5 feet to inches.
   - b) 78 inches to feet.
   - c) 3.5 hours to minutes.
   - d) 360 minutes to hours.
   - e) 2.6 meters to centimeters.
   - f) 3500 meters to kilometers.

SUMMARY:

Why was it necessary to create a code 1 and 2 for this program?

HOMEWORK:

1. Convert . . .
   - a) 115°F to Celsius.
   - b) 36°C to Fahrenheit.
   - c) 100 yards to feet.
   - d) 32 feet to yards.
   - e) 32 feet to inches.
   - f) $2\frac{1}{4}$ hours to minutes.
   - g) 150 minutes to hours.
   - h) 92 pounds to ounces.
   - i) 150 millimeters to centimeters.
   - j) 2.8 centimeters to millimeters.
2. Complete the instructions for the following problem.
   - a) Develop an algorithm.
   - b) Write a plan.
   - c) Develop a flowchart.
   - d) Design a planned output.
   - e) Code a program.

Problem:

Have the computer convert millimeters to centimeters or centimeters to millimeters, user's choice.

HOMEWORK SOLUTIONS:

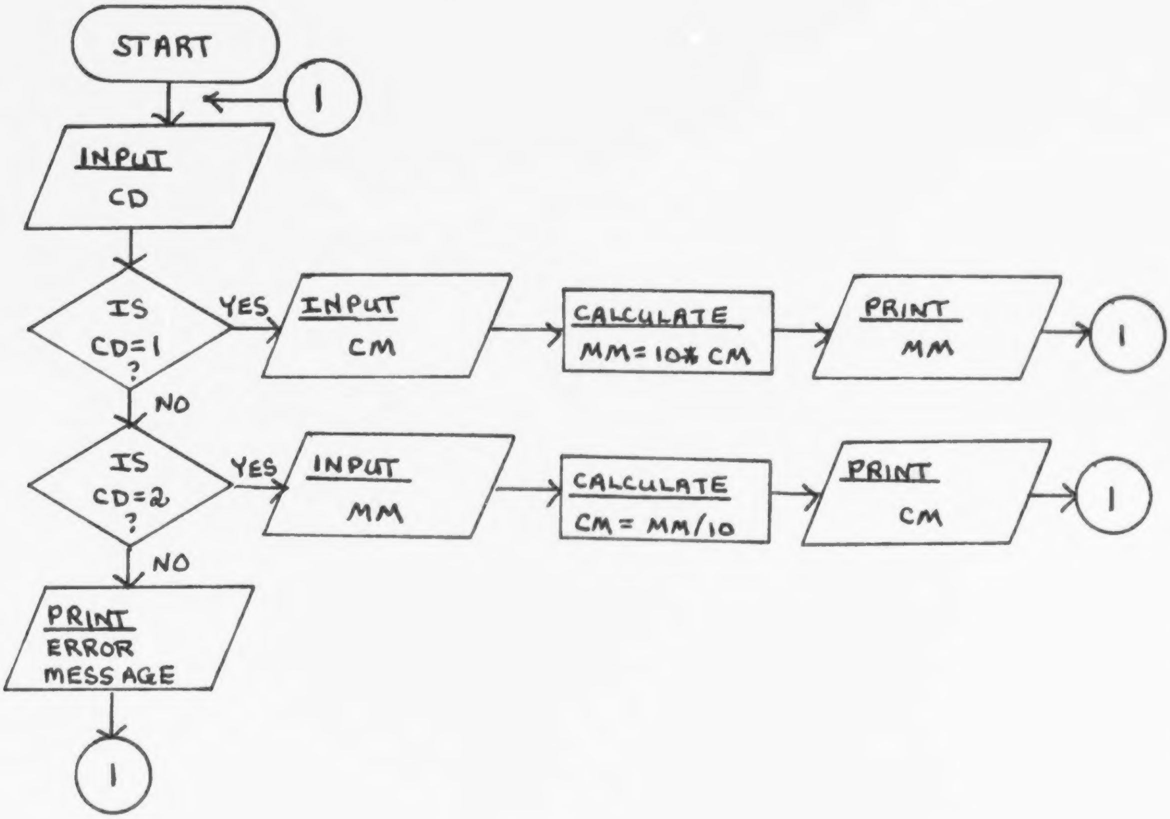
1. a) $46\frac{1}{9}$°C or 46.111...°C b) 96.8°F c) 300 ft.

   d) $10\frac{2}{3}$yd. or 10.66...yd. e) 384 inches f) 135 min.

   g) 2.5 hours h) 1472 oz. i) 15 cm j) 280 mm

2. (A suggested flowchart, planned output, and program are provided. The algorithm and plan can be gleaned from these.)

2c) Flowchart


START
1
INPUT
CD
IS
CD=1
?
YES
INPUT
CM
CALCULATE
MM=10* CM
PRINT
MM
1
NO
IS
CD=2
?
YES
INPUT
MM
CALCULATE
CM = MM/10
PRINT
CM
1
NO
PRINT
ERROR
MESSAGE
1

2d) Planned Output

```
CONVERTING MILLIMETERS AND CENTIMETERS
(Skip 2 lines.)
ENTER 1 FOR CENTIMETERS
AND 2 FOR MILLIMETERS? 3
INCORRECT ENTRY.  PLEASE REENTER.
ENTER 1 FOR CENTIMETERS
AND 2 TO MILLIMETERS? 1
HOW MANY CENTIMETERS? 15
15 CM = 150 MM
(Skip 2 lines.)
ENTER 1 FOR CENTIMETERS
AND 2 FOR MILLIMETERS? 2
HOW MANY MILLIMETERS? 280
280 MM = 28 CM
(Skip 2 lines.)
ENTER 1 FOR CENTIMETERS
AND 2 FOR MILLIMETERS?
```

2e) Program

```
10    PRINT "CONVERTING MILLIMETERS AND CENTIMETERS"
20    PRINT:PRINT
30    PRINT "ENTER 1 FOR CENTIMETERS"
40    INPUT "AND 2 FOR MILLIMETERS";CD
50    IF CD = 1 THEN GOTO 80
60    IF CD = 2 THEN GOTO 110
70    PRINT "INCORRECT ENTRY.  PLEASE REENTER.":GOTO 30
80    INPUT "HOW MANY CENTIMETERS";CM
90    MM=10*CM
100   PRINT CM;"CM = ";MM;" MM":GOTO 20
110   INPUT "HOW MANY MILLIMETERS";MM
120   CM=MM/10
130   PRINT MM;"MM = ";CM;" CM":GOTO 20
140   END
```

Lesson: 12

AIM: How do you solve graduated commission problems?

PERFORMANCE OBJECTIVES: The student will be able to...

1. solve commission problems;
2. solve graduated commission problems.

VOCABULARY: percent, base, gross sales, excess sales, commission, rate of commission, straight commission, graduated commission

MOTIVATION:

Robert was offered a salesman job at Apex Appliance Co. He was given the choice of being paid a wage of $3.50 per hour for 40 hours or 3% commission on all sales. The average salesmen sells $5,000 worth of appliances each week. Which way do you think he should choose to be paid?

DEVELOPMENT:

1. Elicit the students' answers to the motivation problem.
2. Define "commission." Commission is the amount earned calculated as a percent of gross sales. The percent is called the rate of commission.
3. Use the examples in Applications example 1 to review a method for solving percent problems.
4. Elicit a method for solving straight commission problems as follows.
   For each problem, ask the following questions.
   a) What are the gross sales?
   b) What is the rate of commission?
   c) How is the commission calculated?

Problem: Harry sold $550 worth of magazine subscriptions. He was paid 12% commission. How much did he earn?

5. Ask the students to complete Applications example 2.
6. Present to the students the following variation of the motivation problem.

Revised Problem:
The Apex Appliance company hired a new manager who revised the commission as follows:
For the first $4,000 of sales, the rate of commission will be 3%. All subsequent sales will receive a 4% commission.

7. Ask, "Are there more or less earnings for the average salesman?"
8. Define "graduated commission."
9. Use the following problem to elicit a method for finding graduated commission.

Problem: Jean is paid 5% commission on the first $8,000 of sales and 7% on all sales above $8,000. This month she sold $10,000 worth of merchandise. Find the total commission.

a) Calculate the amount of excess sales:
   Excess sales = gross sales - first sales
   Excess sales = 10,000 - 8,000
   Excess sales = $2,000
b) Calculate the commission on the first sales:
   Commission on first sales = first rate x first sales
   Commission on first sales = .05 x 8,000
   Commission on first sales = $400
c) Calculate the commission on the excess sales:
   Commission on excess sales = excess rate x excess sales
   Commission on excess sales =.07 x 2,000
   Commission on excess sales = $140

d) Calculate the total commission:

Total commission = 400 + 140
Total commission = $540

10. Ask the students to complete Applications example 3.

APPLICATIONS:

1. Calculate each.
   a) 25% of $300
   b) 12.5% of $550
   c) 4% of $845.50
   d) $5\frac{1}{3}$% of $6,000
2. Solve each problem.
   a) Michelle made sales of cosmetics totaling $800. She is to be paid a rate of commission of 3.5%. How much did she earn?
   b) If Jason earns a commission of $45 on sales of $900, what is the rate of commission.
3. Calculate the total commission in each.
   a) Howard is paid a graduated commission at the rate of 2% on all sales up to $3,000 and 4% on sales over $3,000. Last week he made sales totaling $5,000. What was his total commission?
   b) Deborah is paid a graduated commission at the rate of 1.5% on the first $2,400 and 3% on the excess. She sold $8,550 worth of computer equipment for the week. What was her total commission?

SUMMARY:

1. What are the advantages and disadvantages for the employee of graduated commission?
2. What are the advantages and disadvantages to for employer of graduated commission?

HOMEWORK:

1. Calculate each.

   a) 35% of 350
   b) 3.2% of 4,500
   c) $66\frac{2}{3}$% of 1,200
   d) 8% of 542.5

2. Solve each problem.

   a) Harriet got a job which pays a salary of $125 plus 5% commission on sales. How much did she earn her first week in which she sold $350 of mercandise?

   b) Sam is working as a door to door salesman of cosmetics and household goods. He is paid a graduated commission at the rate of 2% on the first $500 and 5% on the excess. He sold $850.40 worth of merchandise this week. How much was his total commission?

   c) Jennifer sells clothes in a famous boutique. She is paid $150 plus a graduated commission at the rate of 1% on the first $1,000 and 2.5% on the excess over $1,000. One week she sold $750.00 worth of clothes. In the next week she sold $1,800 worth of clothes. How much did she earn each week?

HOMEWORK SOLUTIONS:

1. a) 122.5   b) 144   c) 800   d) 43.4

2. a) $142.50   b) $27.52   c) first week: $157.50,
   d) second week: $180.00

Lesson: 13

AIM: How do we develop a flowchart solution for a graduated commission problem?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop an algorithm to solve a graduated commission problem;
2. incorporate the algorithm into a plan;
3. flowchart a computer solution.

VOCABULARY: graduated commission, excess sales, rate of commission, first sales

MOTIVATION:

David is paid a graduated commission for the sale of used cars at Honest John's used car lot. He is paid 7.5% on all sales up to $5,000 and 10% on all sales in excess of $5,000. During the first week of the month, David sold three cars for $3,500, $2,850, and $1,550. During the second week, he sold one car for $4,750. What was his commission for each week?

DEVELOPMENT:

1. Elicit responses to the motivation problem. Review and formalize the algorithm for solving graduated commission problems.

   Algorithm

   a) Determine the excess sales, if there are any.
   b) Calculate the commission on the first sales.
   c) If necessary, calculate the commission on the excess sales. Discuss when this calculation is necessary.
   d) Calculate the sum of the two commissions.

   (Note: Point out that algorithms are not always expressed as mathematical formulae.)

2. Ask, "How can we incorporate this algorithm into a plan?"
   a) Ask the students to write a dictionary of variables as follows.

Dictionary

TS = TOTAL SALES
ES = EXCESS SALES
FS = FIRST SALES
FR = RATE OF COMMISSION ON FIRST SALES (IN DECIMAL FORM)
ER = RATE OF COMMISSION ON EXCESS SALES (IN DECIMAL FORM)
CF = COMMISSION ON FIRST SALES
CE = COMMISSION ON EXCESS SALES
CT = TOTAL COMMISSION

   b) State the computer problem as follows.

Problem: Enter the total sales (TS), the first sales (FS), the rate (in decimal form) of commission on the first sales (FR), and the rate (in decimal form) of commission on the excess sales (ER). Have the computer find the total commission (CT).

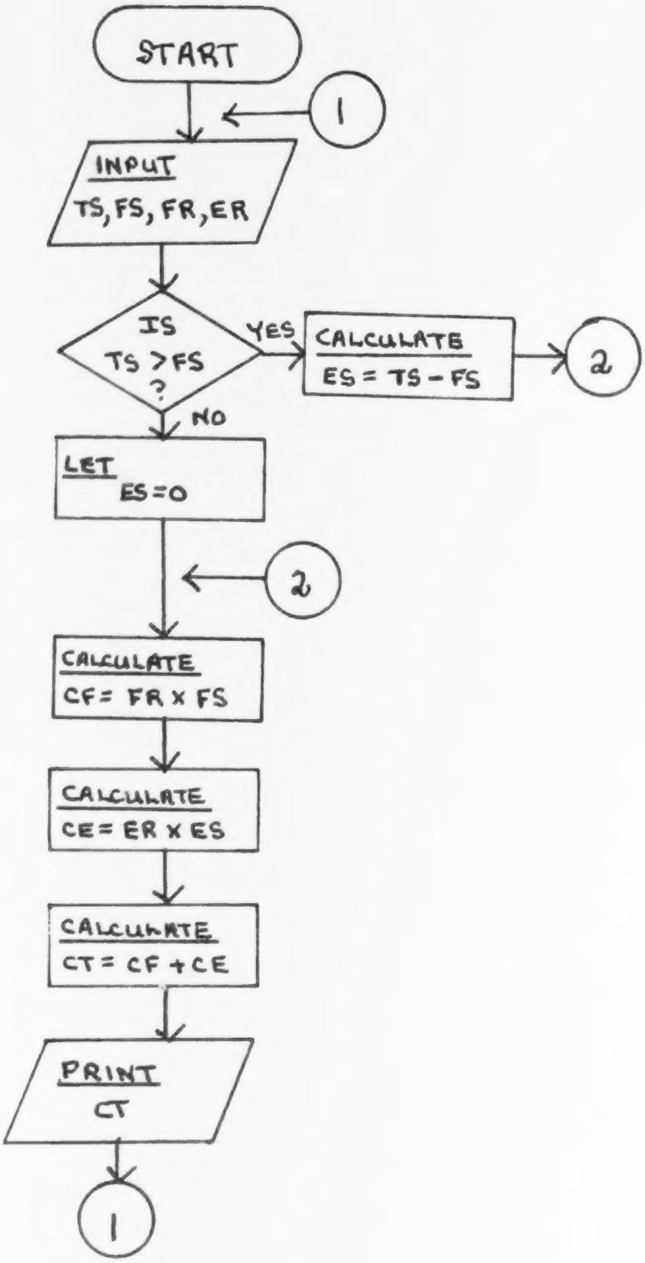
   c) Elicit the plan which follows.

Plan

1) Enter TS, FS, FR, AND ER.
2) Is TS>FS?
   a. If yes, then calculate: ES=TS-FS. Go to 4.
   b. If no, continue.
3) Let ES=0.
4) Calculate: CF=FRxFS.
5) Calculate: CE=ERxES.
6) Calculate: CT=CF+CE.
7) Print the total commission: CT.
8) Go to 1.

3. Elicit the flowchart which follows.

Flowchart


START
1
INPUT
TS, FS, FR, ER
IS
TS > FS
?
YES
CALCULATE
ES = TS − FS
2
NO
LET
ES = 0
2
CALCULATE
CF = FR × FS
CALCULATE
CE = ER × ES
CALCULATE
CT = CF + CE
PRINT
CT
1

4. Ask the students to trace their flowchart in order to determine the commission in the following Applications problems.

APPLICATIONS: Find the total commission for each.

	TS	FS	FR	ER
1)	$2,800	$1,500	2 %	5%
2)	$5,450	$2,500	10%	15%
3)	$1,450	$2,000	5.5%	15%

SUMMARY:

1. Why is it necessary to check if the total sales are greater than the first sales?
2. Why are the excess sales set equal to zero?

HOMEWORK:

Calculate the total commission by tracing the flowchart developed in today's lesson.

1. Find the total commission for $3,450 total sales with a 5% rate of commission on the first $2,000 and 12% rate of commission on the excess.
2. Find the total commission for $12,280 total sales with a 15% rate of commission on the first $10,000 and 25% rate of commission on the excess.
3. Ralph is working for Long Life Insurance Company Incorporated. He is paid 12% commission on sales up to $50,000 and 18% on sales in excess of $50,000. During the year he sold three life insurance policies with face values of $10,000, $25,000 and $40,000. How much total commission did he earn?
4. Pamela is selling magazine subscriptions and is paid a weekly commission of 8% on total sales of $150 and 10% on the excess above $150. She sold subscriptions totaling $225 the first week, $120 the second week, and $350 the third week. How much did she earn each week?

HOMEWORK SOLUTIONS:

1. $274
2. $2,070
3. $10,500
4. Week 1: $19.50, Week 2: $9.60, Week 3: $32.

Lesson: 14L

AIM: How do we write a program to have the computer determine graduated commissions?

PERFORMANCE OBJECTIVES: The student will be able to...

1. design a planned output;
2. write a program from the planned output and the flowchart;
3. incorporate the REM statement in the coding of a program.

VOCABULARY: first sales, excess sales, total sales, rate of commission, total commission

MOTIVATION:

Use of the flowchart developed in Lesson 13 to determine the total commission for . . .

1) $580 total sales with a 3% rate of commission on the first $250 of sales and 5% rate of commission on the excess.
2) $175 total sales with same rates as #1.

DEVELOPMENT:

1. Elicit the solution to the motivation problem and review the algorithm and the vocabulary used in graduated commission problems.
2. Elicit the planned output as follows. (Ask the students to devise their own planned outputs and assign a few to write them on the chalkboard.)

Planned Output

```
CALCULATING GRADUATED COMMISSIONS
(Skip 2 lines.)
ENTER THE TOTAL SALES? 580
ENTER THE FIRST SALES? 250
ENTER, AS A DECIMAL, THE FIRST RATE OF COMMISSION? .03
ENTER, AS A DECIMAL, THE EXCESS RATE OF COMMISSION? .05
(Skip 1 line.)
THE TOTAL COMMISSION IS $ 24
(Skip 2 lines.)
ENTER THE TOTAL SALES? 175
ENTER THE FIRST SALES? 250
ENTER, AS A DECIMAL, THE FIRST RATE OF COMMISSION? .03
ENTER, AS A DECIMAL, THE EXCESS RATE OF COMMISSION? .05
(Skip 1 line.)
THE TOTAL COMMISSION IS $ 5.25
(Skip 2 lines.)
ENTER THE TOTAL SALES?
```

3. Elicit the coded program which follows.
   Introduce the use of REM statements. Stress that REM statements are used for documentation and are not executed when the program is run.

Program

```
10  REM WRITTEN BY (STUDENT'S NAME)
20  REM (ENTER DATE.)
30  REM TS=TOTAL SALES
40  REM FS=FIRST SALES
50  REM FR=RATE OF COMMISSION ON THE FIRST SALES
60  REM ER=RATE OF COMMISSION ON THE EXCESS SALES
70  REM ES=EXCESS SALES
80  REM CF=COMMISSION ON FIRST SALES
90  REM CE=COMMISSION ON EXCESS SALES
100 REM CT=TOTAL COMMISSION
110 PRINT "CALCULATING GRADUATED COMMISSIONS"
120 PRINT:PRINT
130 INPUT "ENTER THE TOTAL SALES";TS
140 INPUT "ENTER THE FIRST SALES";FS
150 INPUT "ENTER, AS A DECIMAL, THE FIRST RATE OF
COMMISSION";FR
160 INPUT "ENTER, AS A DECIMAL, THE EXCESS RATE OF
COMMISSION";ER
170 PRINT
180 IF TS>FS THEN ES=TS-FS:GOTO 200
190 ES=0
200 CF=FR*FS
210 CE=ER*ES
220 CT=CF+CE
230 PRINT "THE TOTAL COMMISSION IS $";CT
240 GOTO 120
250 END
```

4. Ask the students to enter, run and test their programs using the examples from the previous night's homework. Ask those students who complete the program to work on the Application problem.

APPLICATION:

Revise the program to add a bonus of $100 if there are excess sales.

SUMMARY:

What is the purpose of the REM statement in a BASIC program?

HOMEWORK:

Complete the instructions below for the following problem.

a) Write an analysis which includes the development of an algorithm.

b) Write a plan for a computer solution.

c) Develop a flowchart.

d) Design a planned output.

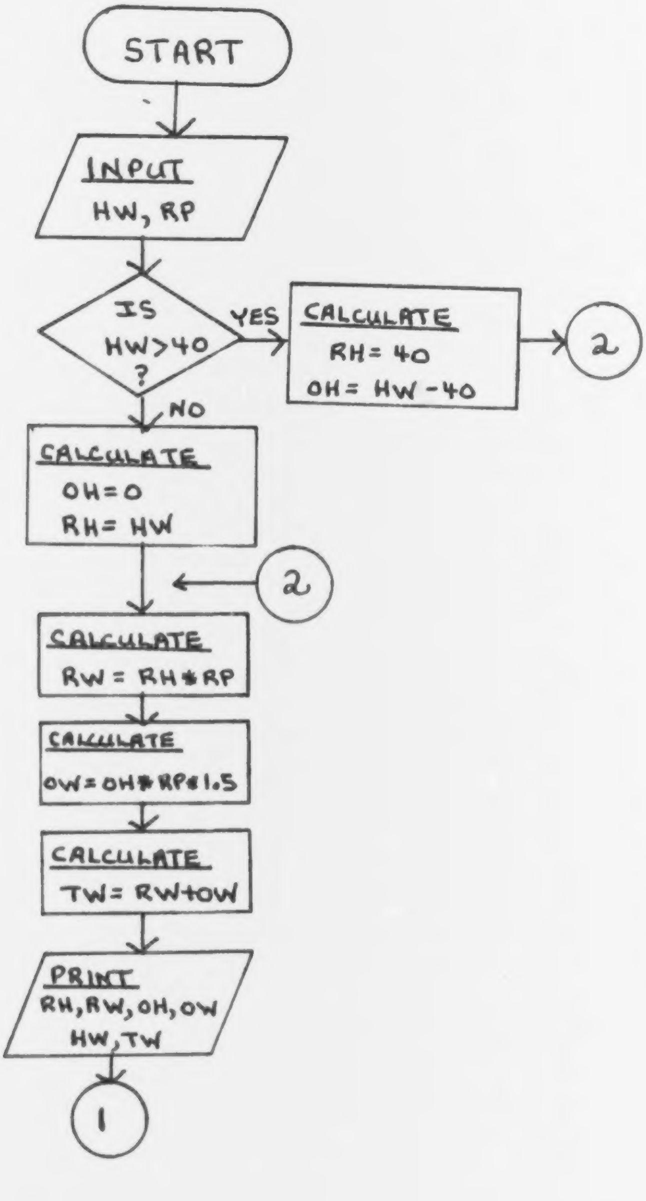
e) Code a program to solve the problem. Be sure to identify, with REM statements, all the variables used.

Problem: An employee of Savmore Supermarket is paid a regular hourly wage for the first 40 hours and time-and-one-half for all additional hours. Have the computer calculate the wages of an employee, after entering the hourly rate of pay and the number of hours worked. Have it display the regular hours and regular wages, the overtime hours and overtime wages, and the total hours and total wages.

HOMEWORK SOLUTIONS:

(A suggested flowchart, planned output and program are provided. The analysis and plan can be gleaned from these.)

c) Flowchart


START
INPUT
HW, RP
IS
HW>40
?
YES
CALCULATE
RH= 40
OH= HW -40
2
NO
CALCULATE
OH=0
RH= HW
2
CALCULATE
RW = RH*RP
CALCULATE
OW=OH*RP*1.5
CALCULATE
TW= RW+OW
PRINT
RH,RW,OH,OW
HW,TW
1

d) Planned Output

```
CALCULATING WAGES
(Skip 2 lines.)
HOW MANY HOURS WORKED? 36
WHAT IS THE RATE OF PAY? 3.75
REGULAR HOURS = 36
REGULAR WAGES = $ 135
OVERTIME HOURS = 0
OVERTIME WAGES = $ 0
TOTAL HOURS = 36
TOTAL WAGES = $ 135
(Skip 2 lines.)
HOW MANY HOURS WORKED? 42
WHAT IS THE RATE OF PAY? 3.75
REGULAR HOURS = 40
REGULAR WAGES = $ 150
OVERTIME HOURS = 2
OVERTIME WAGES = $ 11.25
TOTAL HOURS = 42
TOTAL WAGES = $ 161.25
(Skip 2 lines.)
HOW MANY HOURS WORKED?
```

e) Program

```
10    REM WRITTEN BY ...
20    REM (DATE)
30    REM HW= TOTAL HOURS WORKED
40    REM RH= REGULAR HOURS WORKED
50    REM OH= OVERTIME HOURS WORKED
60    REM RP= RATE OF PAY
70    REM RW= REGULAR WAGES
80    REM OW= OVERTIME WAGES
90    REM TW= TOTAL WAGES
```

```
100  PRINT "CALCULATING WAGES"
110  PRINT:PRINT
120  INPUT "HOW MANY HOURS WORKED";HW
130  INPUT "WHAT IS THE RATE OF PAY";RP
140  IF HW>40 THEN RH=40:OH=HW-40:GOTO 160
150  OH=0:RH=HW
160  RW=RH*RP:OW=OH*RP*1.5
170  TW=RW+OW
180  PRINT "REGULAR HOURS = ";RH
190  PRINT "REGULAR WAGES = $";RW
200  PRINT "OVERTIME HOURS = ";OH
210  PRINT "OVERTIME WAGES = $";OW
220  PRINT "TOTAL HOURS = ";HW
230  PRINT "TOTAL WAGES = $";TW
240  GOTO 110
250  END
```

Lesson: 15

AIM: How do we determine if a given number is divisible by another number?

PERFORMANCE OBJECTIVES: The student will be able to...

1. state the rules for divisibility by 2,3,4,5,6,8,9,10 and 11;
2. apply the divisibility rules to determine if a given number is divisible by 2,3,4,5,6,8,9,10 or 11.

VOCABULARY: divisible, digit, whole number, counting number, sum

MOTIVATION:

By what numbers is 360 divisible?

DEVELOPMENT:

1. Elicit responses to the motivation problem. Discuss the definition of "divisible."

   If dividing a given whole number by a second counting number results in a remainder which equals zero, then the given number is divisible by the second number.

2. Ask, "Can you determine, without dividing, by which number the following numbers are divisible... 28, 342, 1036, 54, 500?"
   Elicit that if the ones' place contains a 0,2,4,6, or 8, then the given number is divisible by 2.

3. Ask, "Can we make a rule for divisibility by 3,4,5,8,9,10 or 11?" Explore with the students numbers divisible by 3,4,5,8,9,10 and 11 and elicit the following rules.

A number is divisible by ...

3, if the sum of its digits is divisible by 3;
4, if the number formed by its last two digits is divisible by 4;
5, if the ones' digit is 0 or 5;
8, if the number formed by its last three digits is divisible by 8;
9, if the sum of its digits is divisible by 9;
10, if the ones' digit is a 0;
11, if the difference of the sums of alternate digits is divisible by 11.

4. Ask, "How can we test to see if a number is divisible by 6?"

   Elicit that if a number is divisible by 2 and 3, then it is divisible by 6.

5. Ask the students to apply these rules to the problems in the Applications.

APPLICATIONS:

1. Use the divisibility rules to determine by which number(s) (2,3,4,5,6,8,9,10,11), if any, each is divisible?

   a) 11,880 b) 92 c) 153 d) 341 e) 2,295 f) 91

2. Using the divisibility rules, determine the value(s) of the digit "d" which will make the number divisible as indicated.
   - a) 53d divisible by 2.
   - b) d841 divisible by 3.
   - c) 560d divisible by 5.
   - d) 8d76 divisible by 9.
   - e) 10d1 divisible by 11.
   - f) 3d6 divisible by 6.
   - g) 2d5120 divisible by 8.

SUMMARY:

How do we determine divisibility by numbers for which we do not have a rule?

HOMEWORK:

1. Use the divisibility rules to determine by which number(s) (2,3,4,5,6,8,9,10,11), if any, each is divisible.

   a) 0 b) 23,760 c) 715 d) 1,836 e) 1,081

2. Using the divisibility rules, determine the value(s) of the digit "d" which makes the number divisible as indicated.
   - a) d481 divisible by 3.
   - b) 601d divisible by 2.
   - c) 54d8 divisible by 4.
   - d) 1d85 divisible by 11.
   - e) 182dd divisible by 9.
   - f) d98 divisible by 6.
   - g) 61515d divisible by 8.

HOMEWORK SOLUTIONS:

1. a) 2,3,4,5,6,8,9,10,11
   b) 2,3,4,5,6,8,9,10,11
   c) 5,11
   d) 2,3,4,6,9
   e) none

2. a) 2,5, or 8
   b) 0,2,4,6, or 8
   c) 0,2,4,6, or 8
   d) 4
   e) 8
   f) 1,4, or 7
   g) 2

Lesson: 16

AIM: How do we develop a flowchart solution to determine if a whole number is divisible by eight?

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate an understanding of the algorithm R=N-Q*8 by incorporating it into a plan for a computer solution;
2. flowchart a computer solution to check divisibility by eight.

VOCABULARY: divisible, divisor, quotient, dividend, remainder

MOTIVATION:

1. Which of the following numbers are divisible by 8?
   a) 96   b) 128   c) 236   d) 108
2. What is the largest number that is less than 1000 which is divisible by 8?

DEVELOPMENT:

1. Elicit responses for the motivation questions.
2. Ask, "What is the rule for divisibility by 8? Without a rule, how can we determine if a number is divisible by another number?"
3. Elicit the algorithm by using the example below.

$$\begin{array}{rrl} \text{Quotient} \rightarrow & 13 & \\ \text{Divisor} \rightarrow \quad 8 & \overline{)108} & \leftarrow \text{Dividend} \\ & \underline{-\ 104} & \\ & 4 & \leftarrow \text{Remainder} \end{array}$$

Ask the students to conclude that the
REMAINDER = DIVIDEND - QUOTIENT * DIVISOR or
R = N - Q * D.

4. Ask the students to apply the algorithm to divisibility by 8.

   Let R = Remainder,
   N = Dividend, and
   Q = Quotient.

   Then, R = N - Q*8.

5. Ask "How can this algorithm be used to test for divisibility by 8?" (Elicit that when R=0, then N is divisible by 8.) Use the algorithm with N=128 and N=236 as follows.

```
For N=128:        16 ← Q              R = N   - Q*8
               8 )128 ← N     or      R = 128-16*8
                - 128                 R = 128-128
                    0 ← R             R = 0
```

Since R=0, 128 is divisible by 8.

```
For N=236:        29 ← Q              R = N   - Q*8
               8 )236 ← N     or      R = 236-29*8
                - 232                 R = 236-232
                    4 ← R             R = 4
```

Since R=4, 236 is not divisible by 8.

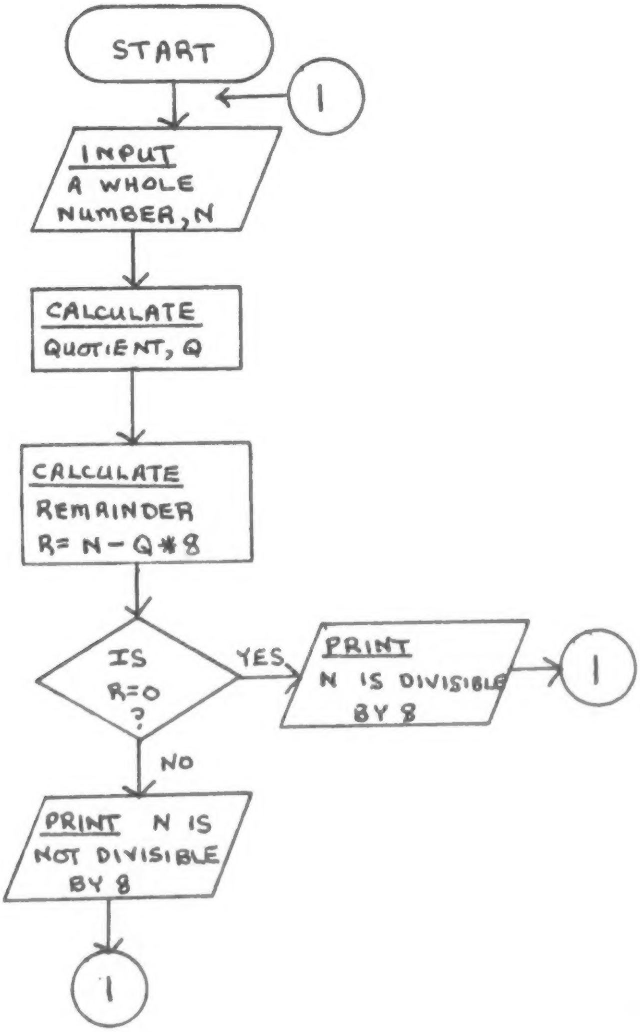
6. Develop the following plan with the students.

   Plan

   1) Enter a whole number, N.
   2) Calculate the quotient, Q.
   3) Calculate the remainder: R = N-Q*8
   4) Is R = 0?
      a) If yes, print N IS DIVISIBLE BY 8. Then go back to 1.
      b) If no, continue.
   5) Print N IS NOT DIVISIBLE BY 8.
   6) Go to 1.

7. Ask the students to write the following flowchart.

Flowchart


START
1
INPUT
A WHOLE
NUMBER, N
CALCULATE
QUOTIENT, Q
CALCULATE
REMAINDER
R= N-Q*8
IS
R=0
?
YES
PRINT
N IS DIVISIBLE
BY 8
1
NO
PRINT N IS
NOT DIVISIBLE
BY 8
1

8. Ask the students to complete the Applications problems.

APPLICATIONS:

1. Using the algorithm R=N-Q*D to calculate the remainder, determine whether...
    a) 258 divisible by 8?
    b) 1069 divisible by 9?
    c) 390 divisible by 15?
    d) 302 divisible by 7?
    e) 546 divisible by 13?
2. Find the smallest number that must be added to 886 to make it divisible by 8.
3. Find the smallest number that must be added to 1005 to make it divisible by 8.

SUMMARY:

1. How can we determine divisibility without using the rules from yesterday's lesson?
2. Are any odd numbers divisible by eight? Why? Why not?
3. If a number is not divisible by four, can it be divisible by eight? Why? Why not?

HOMEWORK:

1. Using the algorithm R=N-Q*D to calculate the remainder, determine whether...
    a) 121 divisible by 7?
    b) 6024 divisible by 12?
    c) 920 divisible by 8?
    d) 900 divisible by 13?
2. Find the smallest number that must be added to 2500 to make it divisible to 8.
3. Using the divisibility tests that you learned yesterday, devise a divisibility test for 24.

HOMEWORK SOLUTIONS:

1. a) 121 is not divisible by 7 because the remainder is 2. (R=121-17*7)
   b) 6024 is divisible by 12 because the remainder is 0. (R=6024-502*12)
   c) 920 is divisible by 8 because the remainder is 0. (R=920-115*8)
   d) 900 is not divisible by 13 because the remainder is 3. (R=900-69*13)
2. The smallest number is 4.
3. In order for a number to be divisible by 24, it must be divisible by 3 (the sum of its digits is divisible by 3) and by 8 (the number formed by the last three digits is divisible by 8).

Lesson: 17L

AIM: How do we code a program that will determine if a number is divisible by eight?

PERFORMANCE OBJECTIVES: The student will be able to...

1. use the "INTeger" function in truncating a number and producing a whole number;
2. incorporate the "INTeger" function in the coding of a program;
3. develop a planned output and code a program to determine divisibility by eight.

VOCABULARY: INT(X), truncate, argument, even, odd

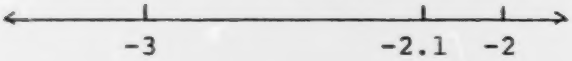
MOTIVATION:

1. Using the algorithm R=N-Q*8 to calculate the remainder, determine whether 206 is divisible by 8?
2. When the computer calculates 206/8, what will it produce? (The students may use the computer to answer this question; e.g., PRINT 206/8.)

DEVELOPMENT:

1. Using the motivation questions, say to the students that when the computer divides two numbers, it produces a complete quotient in decimal form and not the "partial quotient" needed by our algorithm for divisibility by 8; e.g., R=N-Q*8 where Q would be the "partial quotient."
2. In order to obtain the "partial quotient", we use a BASIC function which is a small program that is stored inside the computer. We, the programmers, can use a function by writing the code name for the function in the correct place in the program.
3. The BASIC function that we need here is called the INTeger function. The INTeger function, written INT(X) where X is called the argument of the function, returns the largest integer not greater than the argument X.

Ask, "What would be the value of INT(3.6)? ...INT(21.9)? ...INT(10.1)? ...INT(8.5)? ...INT(5)? ...INT(-2.1)? ...INT(-1.7)?" Ask a student to type into the computer PRINT INT(3.6), PRINT INT(21.9), ... in order to verify the students' responses. Ask the students to note that for positive numbers, the computer truncates the digits to the right of the decimal point. Placing the negative numbers on a number line will help the students to recognize that the INTeger function does not have the same effect on negative numbers; e.g.,



(-2>-2.1)

3. Ask the students to complete Applications problem 1.

4. Ask the students to develop a planned output for the divisibility by 8 problem begun yesterday. One possibility is listed below.

Planned Output

```
DIVISIBILITY BY 8
(Skip 2 lines.)
ENTER AN INTEGER? 206
(Skip a line.)
206 IS NOT DIVISIBLE BY 8.
(Skip 2 lines.)
ENTER AN INTEGER? 72
(Skip a line.)
72 IS DIVISIBLE BY 8.
(Skip 2 lines.)
ENTER AN INTEGER?
```

4. Ask the students to code the program using their own outputs and the flowchart developed in Lesson 16. Circulate around the laboratory to assist those students having difficulty and to give support to the others. Send the students to the computers to enter, run and correct the programs. One possible program follows.

Program

```
10  PRINT "DIVISIBILITY BY 8"
20  PRINT:PRINT
30  INPUT "ENTER AN INTEGER";N
40  PRINT
50  Q=INT(N/8)
60  R=N-Q*8
70  IF R=0 THEN PRINT N;"IS DIVISIBLE BY 8.":GOTO 20
80  PRINT N;"IS NOT DIVISIBLE BY 8.":GOTO 20
90  END
```

6. Ask the students to complete Applications problems 2 and 3.

APPLICATIONS:

1. Find the value of each.

   a) INT(206/8)　　c) INT(1022/8)
   b) INT(72/8)　　d) INT(315/8)

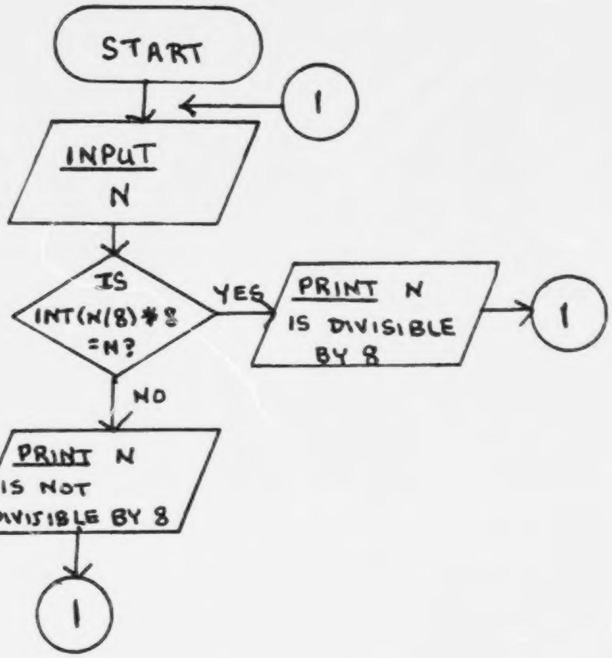
2a) Find the value of each.

   i) INT(206/8)*8　　iii) INT(1022/8)*8
   ii) INT(72/8)*8　　iv) INT(315/8)*8

   b) Is INT(206/8)*8=206?
   c) Is INT(72/8)*8=72?
   d) Under what circumstances is INT(N/8)*8=N?

3a) Using the flowchart which follows, revise the program written in today's lesson to incorporate the algorithm used in question 2.

Flowchart


START
1
INPUT
N
IS
INT(N/8)*8
=N?
YES
PRINT N
IS DIVISIBLE
BY 8
1
NO
PRINT N
IS NOT
DIVISIBLE BY 8
1

3b) RUN your revised program on the computer.

SUMMARY:

1. Without having the computer calculate the remainder first, what algorithm could you use to have the computer check if a number, N, is divisible by 6?
   (Elicit INT(N/6)*6=N.)
2. Without having the computer calculate the remainder first, what algorithm could you use to have the computer check if a number, N, is even or odd?
   (Elicit INT(N/2)*2=N.)

HOMEWORK:

For the following problem, complete the instructions.

a) Develop a flowchart.

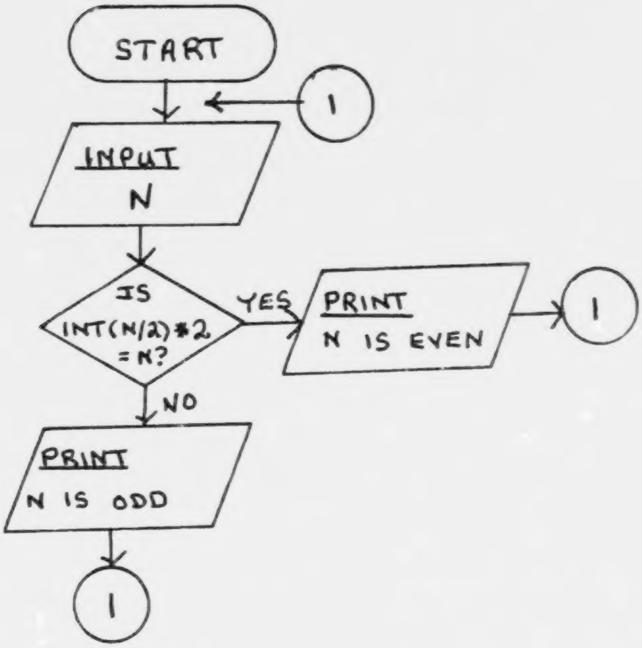
b) Design an output.

c) Code a program that will solve the problem.

Problem: Enter a positive integer. Have the computer determine whether it is even or odd.

HOMEWORK SOLUTIONS:

a) Flowchart


START
1
INPUT
N
IS
INT(N/2)*2
=N?
YES
PRINT
N IS EVEN
1
NO
PRINT
N IS ODD
1

b) Planned Output

```
EVEN OR ODD?
(Skip 2 lines.)
ENTER A POSITIVE INTEGER? 207
(Skip a line.)
207 IS ODD.
(Skip 2 lines.)
ENTER A POSITIVE INTEGER? 250
(Skip a line.)
250 IS EVEN.
(Skip 2 lines.)
ENTER A POSITIVE INTEGER?
```

c) Program

```
10  PRINT "EVEN OR ODD?"
20  PRINT:PRINT
30  INPUT "ENTER A POSITIVE INTEGER";N
40  PRINT
50  IF INT(N/2)*2=N THEN PRINT N;"IS EVEN.":GOTO 20
60  PRINT N;"IS ODD.":GOTO 20
70  END
```

Lesson: 18L

AIM: How can we write a program that will separate the digits of a two-digit natural number?

PERFORMANCE OBJECTIVES: The student will be able to...

1. separate the digits of a two-digit natural number by using an algorithm;
2. incorporate the algorithm into a plan;
3. flowchart a computer solution for separating the digits;
4. write a program from the flowchart and a planned output.

VOCABULARY: natural number, truncate, digit, numeral, tens' digit, units' digit

MOTIVATION:

When we look at the numeral 29, our eye can separate it into the digits ... tens' digit, 2 and units' digit, 9. Since the computer does not have eyes, what operations can we use to separate the digits of the numeral 29?

DEVELOPMENT:

1. Elicit the following algorithm as a response to the motivation problem.
   a) Divide the number by 10; e.g., 29/10 = 2.9.
   b) Truncate the digits of the quotient to the right of the decimal point leaving the integer part of the number. This produces the tens' digit; e.g., INT(2.9) = 2.
   c) Multiply the tens' digit by 10 and subtract the product from the original number, thus producing the units' digit 29-2x10=9.
2. Ask the students to verify this algorithm using other two-digit natural numbers.

3. Ask, "How can this algorithm be incorporated into a plan for a computer solution for separating the digits of a two-digit natural number?"
    a) Ask the students to prepare a dictionary listing the variables that will be used in the plan.

        Dictionary

        N = a two-digit natural number
        T = tens' digit
        U = units' digit

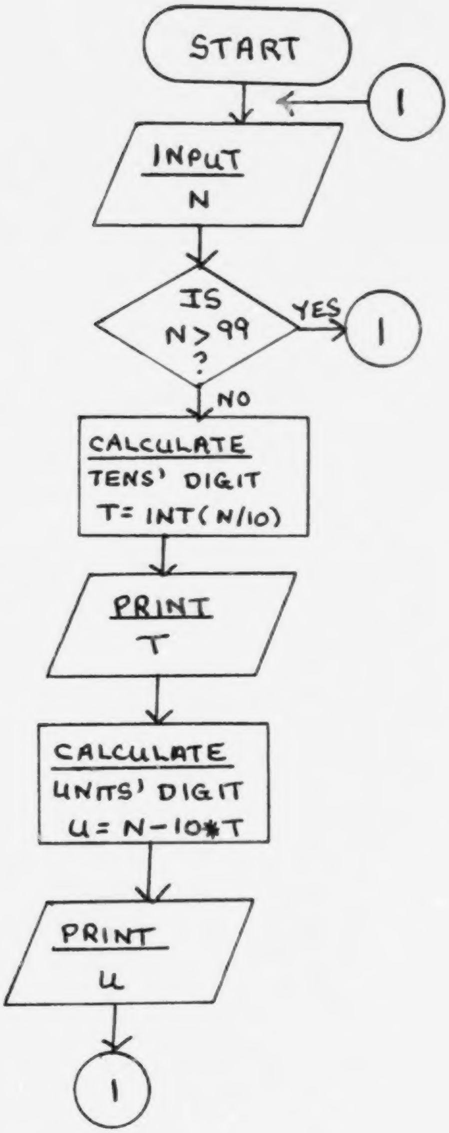
    b) Elicit the plan which follows.

    Plan

    1) Enter a two-digit natural number, N.
    2) Is N>99?
        a) If yes, print an appropriate message and then go back to 1.
        b) If no, continue.
    3) Calculate the value of the tens' digit: T=INT(N/10).
    4) Print T.
    5) Calculate the value of the units' digit: U=N-10xT.
    6) Print U.
    7) Go to 1.

4. Ask the students to develop the flowchart which follows.

Flowchart


START
1
INPUT
N
IS
N>99
?
YES
1
NO
CALCULATE
TENS' DIGIT
T= INT(N/10)
PRINT
T
CALCULATE
UNITS' DIGIT
U= N-10*T
PRINT
U
1

5. Ask the students to design a planned output similar to the one which follows.

Planned Output

```
SEPARATING THE DIGITS
(Skip 2 lines.)
ENTER A TWO-DIGIT NATURAL NUMBER? 68
(Skip 2 lines.)
THE TENS' DIGIT IS 6.
THE UNITS' DIGIT IS 8.
(Skip 2 lines.)
ENTER A TWO-DIGIT NATURAL NUMBER? 134
IT IS NOT A TWO-DIGIT NUMBER!
ENTER A TWO-DIGIT NATURAL NUMBER?
```

6. Ask the students to code, enter and run the program.

Program

```
10  PRINT "SEPARATING THE DIGITS"
20  PRINT : PRINT
30  INPUT "ENTER A TWO-DIGIT NATURAL NUMBER";N
40  IF N > 99 THEN PRINT "IT IS NOT A TWO-DIGIT
NUMBER!":GOTO 30
50  T=INT(N/10)
60  PRINT : PRINT
70  PRINT "THE TENS' DIGIT IS";T;"."
80  U = N - 10 * T
90  PRINT "THE UNITS' DIGIT IS";U;"."
100 GOTO 20
110 END
```

APPLICATIONS:

1. Write each as an expanded numeral.
   a. 68   b. 23   c. 234   d. 109
2. For each, write the number with the digits reversed in both standard and expanded form.
   a. 21   b. 80   c. 259   d. 88
3. Revise the flowchart and program developed in todays' lesson so that the computer will display the two-digit natural number entered, with its digits reversed.

SUMMARY:

1. What algorithm can we use to separate the digits of a three-digit natural number? (See the homework assignment.)
2. In the program that we wrote in today's lesson, what digits will be displayed if the number 7 is entered?

HOMEWORK:

For the following problem, complete the instructions.

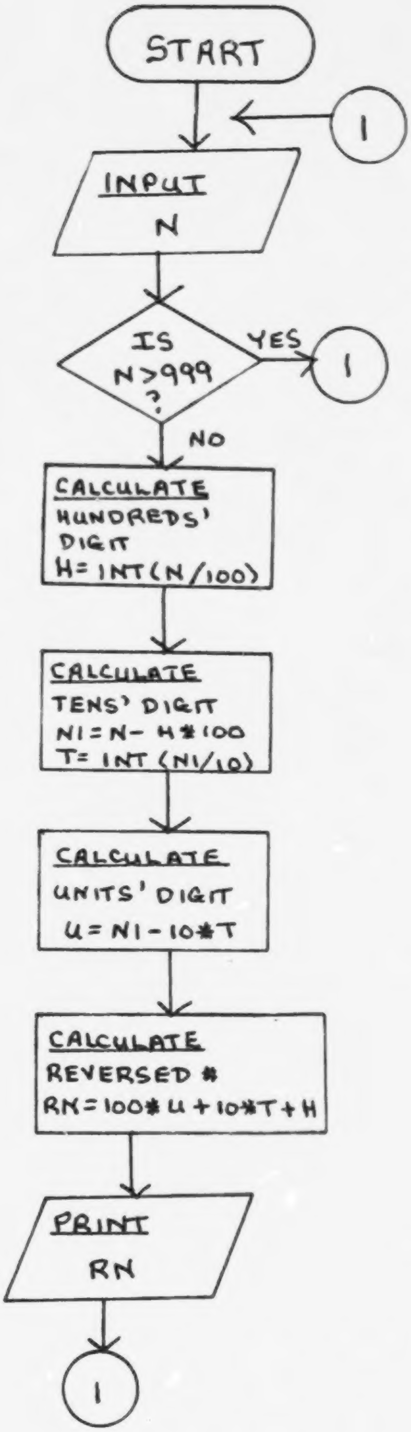
a) Write an analysis which includes the development of an algorithm.
b) Write a plan showing how to use the algorithm to solve the problem.
c) Develop a flowchart.
d) Design a planned output.
e) Code a program.

Problem: Enter a three-digit natural number. Have the computer separate its digits and display the number with the digits reversed.

HOMEWORK SOLUTIONS:

(A possible flowchart, planned output and program follow. The required analysis and plan can be gleaned from these.)

c) **Flowchart**


START
1
INPUT
N
IS
N>999
?
YES
1
NO
CALCULATE
HUNDREDS'
DIGIT
H= INT(N/100)
CALCULATE
TENS' DIGIT
N1=N- H*100
T= INT (N1/10)
CALCULATE
UNITS' DIGIT
U=N1-10*T
CALCULATE
REVERSED #
RN=100*U+10*T+H
PRINT
RN
1

d) Planned Output

```
SEPARATING THE DIGITS
(Skip 2 lines.)
ENTER A THREE-DIGIT NUMBER? 123
(Skip 2 lines.)
THE HUNDREDS' DIGIT IS 1.
THE TENS' DIGIT IS 2.
THE UNITS' DIGIT IS 3.
(Skip 1 line.)
THE NUMBER WITH ITS DIGITS REVERSED IS 321.
(Skip 2 lines.)
ENTER A THREE-DIGIT NUMBER? 58
(Skip 2 lines.)
THE HUNDREDS' DIGIT IS 0.
THE TENS' DIGIT IS 5.
THE UNITS' DIGIT IS 8.
(Skip 1 line.)
THE NUMBER WITH ITS DIGITS REVERSED IS 850.
(Skip 2 lines.)
ENTER A THREE-DIGIT NUMBER? 7
(Skip 2 lines.)
THE HUNDREDS' DIGIT IS 0.
THE TENS' DIGIT IS 0.
THE UNITS' DIGIT IS 7.
(Skip 1 line.)
THE NUMBER WITH ITS DIGITS REVERSED IS 700.
(Skip 2 lines.)
ENTER A THREE-DIGIT NUMBER? 1324
ENTER A THREE-DIGIT NUMBER?
```

e) Program

```
10  PRINT "SEPARATING THE DIGITS"
20  PRINT:PRINT
30  INPUT "ENTER A THREE-DIGIT NATURAL NUMBER";N
40  IF N > 999 THEN GOTO 30
50  H=INT(N/100)
60  PRINT:PRINT:PRINT "THE HUNDREDS' DIGIT IS";H;"."
70  N1 = N - H * 100
80  T=INT(N1/10)
90  PRINT "THE TENS' DIGIT IS";T;"."
100 U = N1 - T * 10
110 PRINT "THE UNITS' DIGIT IS ";U;"."
120 RN=100*U+10*T+H
130 PRINT:PRINT "THE NUMBER WITH ITS DIGITS REVERSED
IS";RN;"."
140 GOTO 20
150 END
```

Lesson: 19

AIM: How do we code a program to have the computer display the sequence of the first fifteen square numbers?

PERFORMANCE OBJECTIVES: The student will be able to...

1. determine the output in a program involving the counter algorithm;
2. code a loop to count from a given whole number to a another given whole number by a given whole number increment;
3. identify "square numbers";
4. flowchart a solution for displaying the first fifteen square numbers;
5. plan and code a program to display the first fifteen square numbers.

VOCABULARY: initialize, step up, step down, counter, termination, assignment, loop, square numbers, cube numbers

MOTIVATION:

Determine the output for the following program.

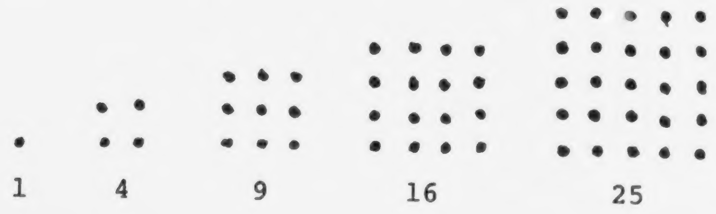
Program

```
10  K=1
20  PRINT K
30  K=K+1
40  IF K>10 THEN GOTO 60
50  GOTO 20
60  END
```

DEVELOPMENT:

1. Elicit responses to the motivation problem. Ask, "What does the program do? What title would you give this program?" Elicit that the program "counts" from one through ten; thus, COUNTER would be a good title.

2. Ask, "How can we change this program so that the computer "counting" starts at 2 and ends at 18?" Discuss "initialization" and "termination."
3. Ask, "How can we change this program to count by twos? Discuss the concept of stepping up the count with the expressions K=K+1 and K=K+2. Remind the students that this is not an algebraic equation but an assignment statement. Ask the students to say K becomes K+1 and K becomes K+2. Say to them that for K=K+1, the value in memory cell K becomes one larger each time the instruction is executed and that value is stored back in memory cell K. Ask, "What happens each time the instruction K=K+2 is executed? What is the purpose of line 40 in the program listed in the motivation? What would happen if line 40 is deleted? What is the purpose of line 10 in the program?"
4. Assign Applications example 1.
5. Ask the students to identify the next number in the sequence of square numbers.



Ask, "How is each square number obtained?" Elicit that a square number is obtained by multiplying a counting number by itself as shown.

Counting number:	1	2	3	4	5	6
	1x1=	2x2=	3x3=	4x4=	5x5=	6x6=
Square number:	1	4	9	16	25	36

6. Present the following problem.

Problem:
Have the computer display the sequence of the first fifteen square numbers.

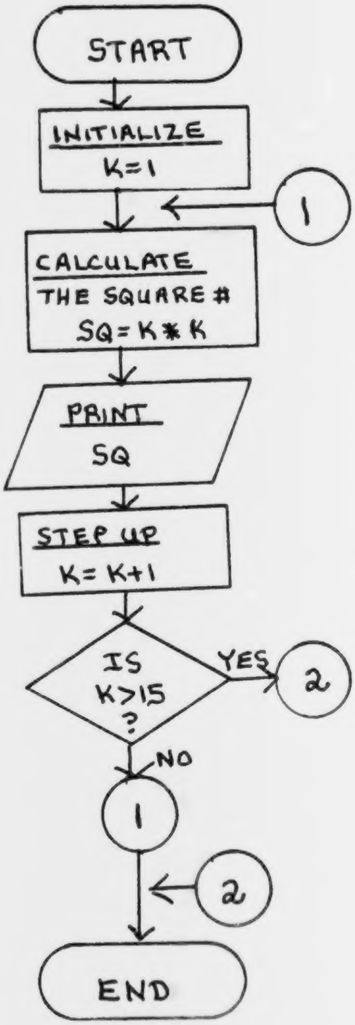
7. Elicit the plan as follows.

Plan

1) Initialize the counter: K=1.
2) Calculate each square number.
3) Print each square number.
4) Step up K by 1.
5) Check: Is K>15?
    a) If yes, go to 7.
    b) If no, continue.
6) Go to 2.
7) End.

8. Elicit the following flowchart.

Flowchart


START
INITIALIZE
K=1
1
CALCULATE
THE SQUARE #
SQ=K*K
PRINT
SQ
STEP UP
K=K+1
IS
K>15
?
YES
2
NO
1
2
END

9. Ask the students to design a planned output; e.g.,

<u>Planned Output</u>

```
SQUARE NUMBERS
(Skip a line.)
1  4  9  16  25 ...
```

10. Elicit the program which follows.

<u>Program</u>

```
10  PRINT "SQUARE NUMBERS"
20  PRINT
30  K=1
40  SQ=K*K
50  PRINT SQ;
60  K=K+1
70  IF K>15 THEN GOTO 90
80  GOTO 40
90  END
```

APPLICATIONS:

1. Show the output for each of the following programs.

a)

```
10  X=3
20  PRINT X
30  X=X+3
40  IF X>15 THEN GOTO 60
50  GOTO 20
60  END
```

b)

```
  10  W=10
  20  PRINT W;
* 30  W=W-1
  40  IF W<1 THEN GOTO 60
  50  GOTO 20
  60  PRINT "BLAST OFF!"
  70  END
```

* Ask the students to note that in this program the computer steps down.

2. Write a counter program so that the computer "counts" from ten through one hundred by tens.

3. In each, determine the square number, given the number of dots on one side of a square.
   a) 3   b) 12   c) 7   d) 25

SUMMARY:

1. What four statements are necessary in the counter algorithm?
2. Why is the counter sometimes described as a "loop"?

HOMEWORK:

1. Show the output for each of the following programs.

a)

```
10  C=1
20  PRINT C;
30  C=C+2
40  IF C>13 THEN 60
50  GOTO 20
60  END
```

b)

```
10  Y=50
20  PRINT Y;
30  Y=Y-5
40  IF Y<10 THEN 60
50  GOTO 20
60  END
```

2. Write a counter program so that the computer counts from three through thirty-three counting by threes.
3. For the following problem, complete the instructions.
   - a) Write an analysis which includes the development of an algorithm.
   - b) Write a plan incorporating the algorithm.
   - c) Develop a flowchart.
   - d) Design a planned output.
   - e) Code a program.

Problem:
Have the computer display the first fifteen cube numbers.

HOMEWORK SOLUTIONS:

1a) Screen

```
RUN
1   3   5   7   9   11   13
```

Memory

C

~~1~~
~~3~~
~~5~~
~~7~~
~~9~~
~~11~~
~~13~~
15

1b) Screen

```
RUN
50  45  40  35  30  25  20  15  10
```

Memory

Y

~~50~~

~~45~~

~~40~~

~~35~~

~~30~~

~~25~~

~~20~~

~~15~~

~~10~~

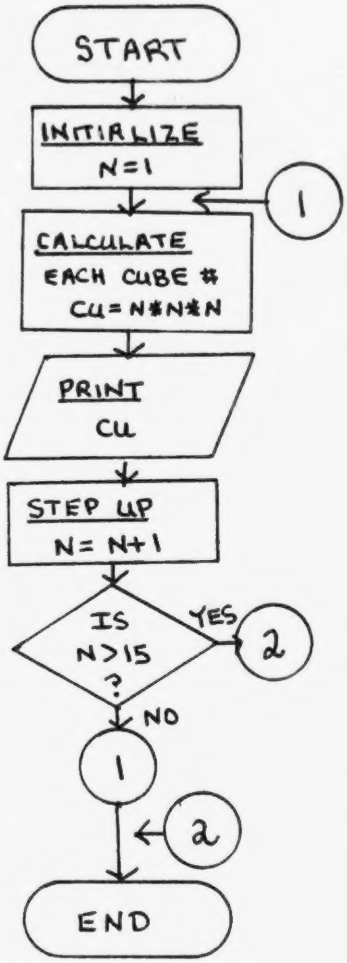
5

2. Program

```
10  K=3
20  PRINT K;
30  K=K+3
40  IF K>33 THEN GOTO 60
50  GOTO 20
60  END
```

3. (A suggested flowchart, planned output and program follow. The analysis and plan can be gleaned from these.)

c) Flowchart


START
INITIALIZE
N=1
1
CALCULATE
EACH CUBE #
CU=N*N*N
PRINT
CU
STEP UP
N= N+1
IS
N>15
?
YES
2
NO
1
2
END

d) Planned Output

```
CUBE NUMBERS
(Skip a line.)
1  8  27  64  125 ...
```

e) Program

```
10  PRINT "CUBE NUMBERS":PRINT
20  N=1
30  CU=N*N*N
40  PRINT CU;
50  N=N+1
60  IF N>15 THEN GOTO 80
70  GOTO 30
80  END
```

Lesson: 20L

AIM: How do we code a program to print a chart showing the areas of triangles each of whose bases has length twelve and whose heights are integers from four through ten inclusive?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to print areas of triangles using the idea of a counter;
2. enter, run, and correct his/her program;
3. state the four fundamental parts of a counter.

VOCABULARY: counter, initialize, step

MOTIVATION:

1. Find the areas of the triangles assigned to your group. (Assign the following sets to different groups of students.)

Set A

1) base = 12
height = 4

2) base = 12
height = 5

Set B

1) base = 12
height = 6

2) base = 12
height = 7

Set C

1) base = 12
height = 8

2) base = 12
height = 9

Set D

1) base = 12
height = 8

2) base = 12
height = 10

2. Ask, "Which quantity changes among the different exercises?" Elicit that the heights change from 4 through 10.

DEVELOPMENT:

1. Say to the students that today they will write a program to have the computer print a chart showing the areas of all triangles with bases 12 and heights from 4 through 10. Write the problem on the chalkboard as follows.

Problem: Have the computer print a chart showing the areas of triangles each of whose bases has length 12 and whose heights are integers from 4 through 10 inclusive.

2. Formulate the following portion of the planned output from the group responses to the motivation.

BASES	HEIGHTS	AREAS
12	4	24
12	5	30
12	6	36
12	7	42
12	8	48
12	9	54
12	10	60

3. Ask, "What new concept learned yesterday will be useful for solving this problem?" Elicit the counter algorithm.

4. Elicit the following analysis from the students.

   Analysis

   a) Ask the students to notice that there are no question marks in the chart. Ask, "Which BASIC statement will produce a question mark when data is entered? How do we get the data entered without question marks appearing?" Say that in this program, neither an INPUT nor a READ-DATA instruction will be used. Instead, a counter will be used.

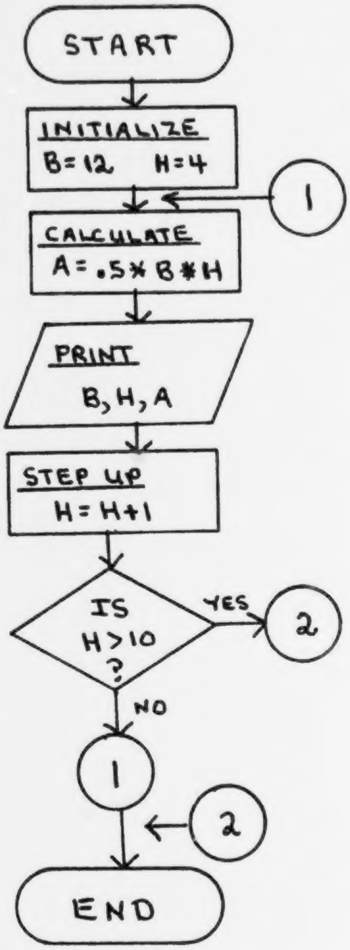
   b) Say that since when we count we must begin somewhere, we will "initialize" the value in memory cell B at 12 and the value in memory cell H at 4. Say to the students that to "initialize" is to give a starting value to a variable in the same way that your initial gives a starting letter to your name.

   c) Ask, "After finding the area, what do we want the computer to do with the value stored in memory cell B?...memory cell H?" Elicit that we want the computer to keep the value in memory cell B the same but to "step" up the value in memory cell H by adding 1 to the previous value in H.

   d) Ask, "When should the computer stop adding 1 to the value in memory cell H; in other words, when should the computer stop 'counting'?" Elicit that when the value in memory cell H is larger than 10, the computer will have counted, displayed and calculated with the whole number values from 4 through 10.

5a) Ask the students to construct the flowchart which follows.

Flowchart


START
INITIALIZE
B=12 H=4
1
CALCULATE
A=.5*B*H
PRINT
B,H,A
STEP UP
H=H+1
IS
H>10
?
YES
2
NO
1
2
END

5b) As the students develop the flowchart, ask the following questions.

i) What constant value do we want to assign to memory cell B? At what value do we want to initialize our count in memory cell H?

ii) What algorithm do we want the computer to use in order to calculate the area of each triangle?

iii) Which values do we want the computer to display? In which memory cells are they located?

iv) How do we get the computer to step up to the next integer value?

v) What check do we want the computer to make so that it will know when to stop counting? What do we want the computer to do if the count is still not greater than 10?...is greater than 10?

5c) Ask, "Which steps form the counter?" Write the four elements of the counter on the chalkboard as follows.

	GENERALLY	IN THIS FLOWCHART
i)	Initialize the variable that counts.	H=4
ii)	Have the computer do what the problem asks with the variable; calculate, print,...	A=.5*B*H PRINT B,H,A
iii)	Step up (or down) the variable that counts.	H=H+1
iv)	Check: Is the value of the variable larger than you want? If no, go back to step ii; otherwise, stop.	IS H>10?

APPLICATIONS:

Complete the instructions which follow.

1) Design your own output by expanding on the chart used in this lesson.
2) Write a program to solve the problem discussed in this lesson.
3) RUN the program.

One possible program is listed below.

Program

```
10     PRINT "AREAS OF TRIANGLES WITH BASE 12 AND"
20     PRINT "VARYING HEIGHTS FROM 4 THROUGH 10"
30     REM THIS PROGRAM USES A COUNTER
40     PRINT "BASES", "HEIGHTS", "AREAS"
50     PRINT "-----", "-------", "-----"
60     B = 12
70     H = 4
80     A = .5*B*H
90     PRINT B, H, A
100    H = H+1
110    IF H>10 THEN GOTO 130
120    GOTO 80
130    END
```

SUMMARY:

What are the four stages of a counter? (INITIALIZE, CALCULATE..., STEP UP, CHECK)

HOMEWORK:

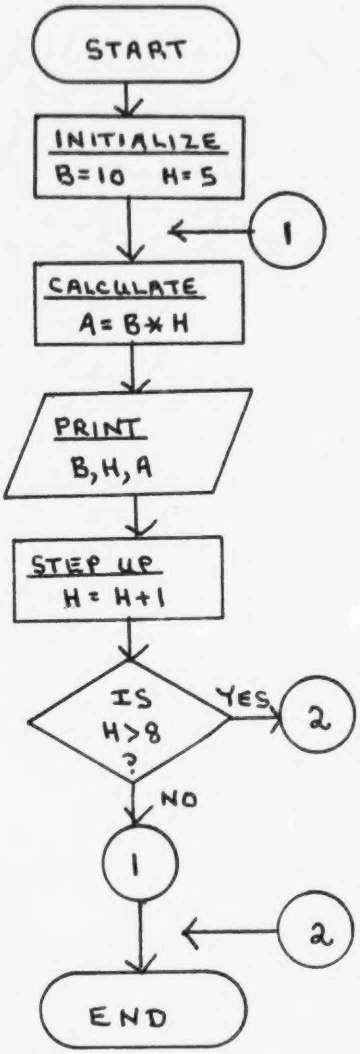
1. What are the four stages of a counter?
2. Rewrite the program from today's lesson so that it will produce the output which follows.

```
B          H          A
60         10         300
70         10         350
80         10         400
90         10         450
100        10         500
```

3. For the given problem, flowchart and planned output, code a program that will solve the problem.

Problem: Have the computer find and display, in chart form, the areas of all rectangles each of whose bases has length ten and whose heights are integers ranging from five through eight.

Flowchart


START
INITIALIZE
B=10 H=5
1
CALCULATE
A=B*H
PRINT
B,H,A
STEP UP
H=H+1
IS
H>8
?
YES
2
NO
1
2
END

Planned Output

```
THE AREAS OF RECTANGLES
B           H          A
10          5          50
10          6          60
10          7          70
10          8          80
```

HOMEWORK SOLUTIONS:

1. a) Initialize.
   b) Calculate...
   c) Step up (or down).
   d) Check...

2.

```
10     PRINT "B", "H", "A"
20     B = 60
30     H = 10
40     A = .5*B*H
50     PRINT B, H, A
60     B = B+10
70     IF B>100 THEN 90
80     GOTO 40
90     END
```

3.

```
10     PRINT "THE AREAS OF RECTANGLES"
20     PRINT "B", "H", "A"
30     B = 10
40     H = 5
50     A = B*H
60     PRINT B, H, A
70     H = H+1
80     IF H>8 THEN GOTO 100
90     GOTO 50
100    END
```

Lesson: 21

AIM: How do we code a program to have the computer display the sequence of the first fifteen triangular numbers?

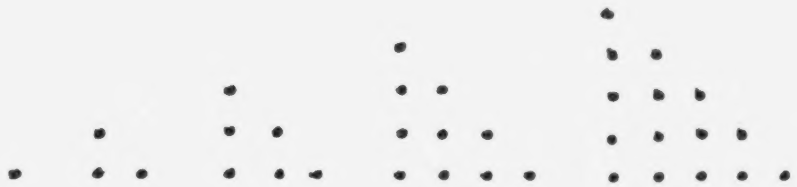
PERFORMANCE OBJECTIVES: The student will be able to...

1. define "triangular numbers";
2. find the n^{th} triangular number;
3. write a program to find the first fifteen triangular numbers.

VOCABULARY: triangular numbers, accumulator, sequence, pentagonal numbers, square numbers, rectangular numbers

MOTIVATION:

1. Draw the next two diagrams in this sequence.



2. Fill in the following chart.

Number of Dots in the "Base"	Number of Dots in the "Triangle"
1	
2	
3	
4	
5	

DEVELOPMENT:

1. Elicit responses to the motivation problems. Ask, "What would be the next line in the chart? How did you get it?"
2. Say to the students that the numbers 1,3,6,10,15,...are called "triangular numbers." Ask, "Can you guess why?" Elicit that they represent the number of dots required to make triangular arrays of equally spaced dots.

3. Say that today we want to have the computer find and display the first fifteen triangular numbers. Present the following problem for a computer solution.

Problem: Have the computer find and display the first fifteen triangular numbers.

Analysis

a) This program will use a counter, C, for the number of dots in the base.
   Ask, "What values will C take on in this program?"
   (C = 1,2,3,...,15)

b) We will call the corresponding triangular number, T.
   Ask, "What will be the initial value of T?" (1)

c) Note that in this program, both C and T change.
   Ask, "How will we change C to get the next value of C?"
   (C = C+1)
   Ask, "How will we change T to get the new value of T?"
   (T = T+C)

In order to assist the students in responding to these questions, you may wish to use the chart already constructed in the motivation as follows.

Number of Dots in the "Base"	Number of Dots in the "Triangle"
1	1
2 = 1+1	3 = 1+2
3 = 2+1	6 = 3+3
4 = 3+1	10 = 6+4
5 = 4+1	15 = 10+5
6 = 5+1	21 = 15+6
(C = C+1)	**(T = T+C)**

d) Ask the students to note that the new value of T must be calculated after the new value of C is found. Ask, "What would happen if we reversed the order and calculated
   T = T+C
   C = C+1 in that order?"

e) Ask, "When should the computer stop calculating new values for the chart?" (when C>15)

4. Develop the following plan with your students.

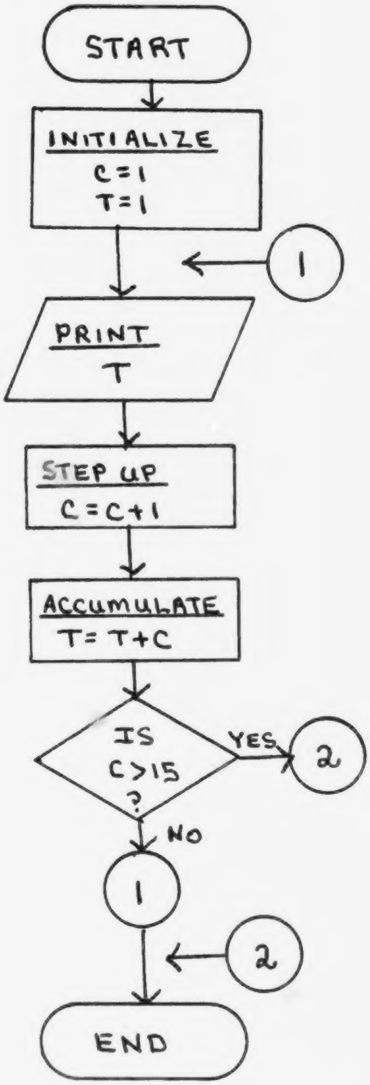
   Plan

   1) Initialize the number of dots in the base: C=1.
   2) Initialize the first triangular number: T=1.
   3) Display each triangular number, T.
      (Ask the students to note that the value of C is not displayed.)
   4) Step up the value in memory cell C: C = C+1.
   5) Add (accumulate) the value in memory cell C to the value in memory cell T: T = T+C.
   6) Check: Is C>15?
      i) If yes, go to 8.
      ii) If no, continue.
   7) Go back to 3.
   8) End.

   Tell the students that memory cell C, along with the appropriate algorithm, serves as a counter. Ask, "Which steps follow the four basic elements of a counter?" Say to the students that since we keep adding numbers to memory cell T and storing the answer back in memory cell T, we call this variable an accumulator.

5. Ask the students to develop a flowchart as follows.

Flowchart


START
INITIALIZE
C=1
T=1
1
PRINT
T
STEP UP
C=C+1
ACCUMULATE
T=T+C
IS
C>15
?
YES
2
NO
1
2
END

6. Ask the students to design an output.

Planned Output

```
TRIANGULAR NUMBERS
1
3
6
10
15
etc.
```

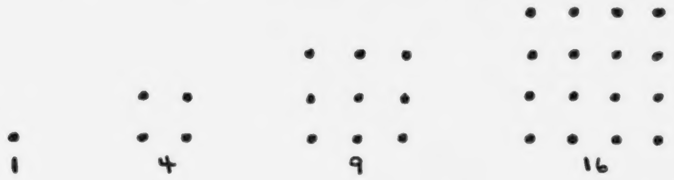
7. Ask the students to code the program which follows.

Program

```
10  PRINT "TRIANGULAR NUMBERS"
20  C = 1
30  T = 1
40  PRINT T
50  C = C+1
60  T = T+C
70  IF C>15 THEN GOTO 90
80  GOTO 40
90  END
```

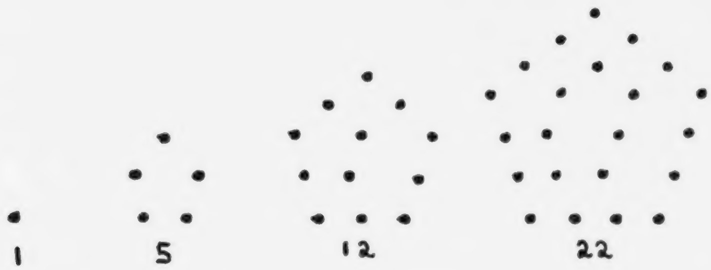
APPLICATIONS:

1a. What are the next two numbers in the sequence?



1b. What is this sequence of numbers called? Why?

2a. What are the next two numbers in the sequence?



2b. What do you think this sequence of numbers is called? Why?

3. Examine the table below.

Triangular Numbers		1	3	6
Square Numbers	1	4	9	16
Pentagonal Numbers	1	5	12	22

a. Add two more columns to the table.

b. Fill in the blanks.

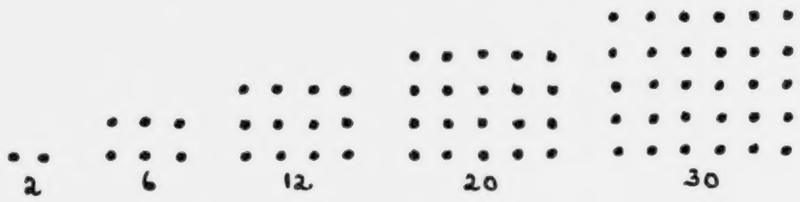
i. The second pentagonal number is the sum of the _____ triangular number and the _____ square number.

ii. The third pentagonal number is the sum of the _____ triangular number and the _____ square number.

iii. The fourth pentagonal number is the sum of the _____ triangular number and the _____ square number.

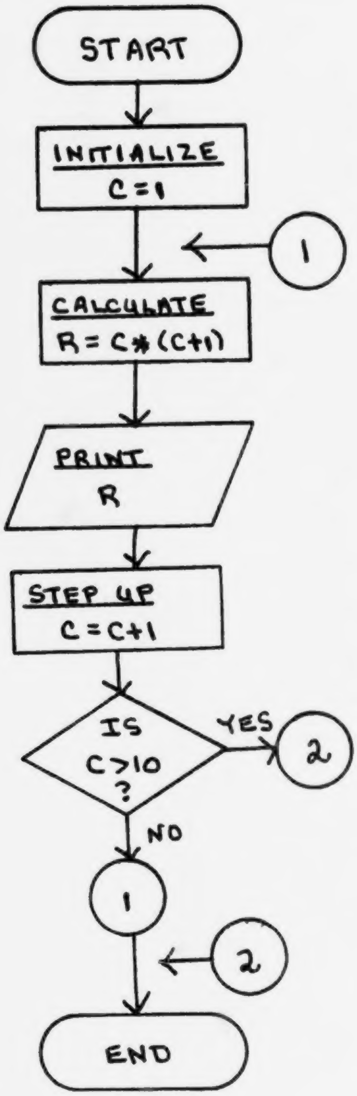
iv. The n^{th} pentagonal number is the sum of the ____ triangular number and the _____ square number.

4. These numbers are called rectangular numbers.



Using the flowchart and planned output which follow, code a program that will have the computer display the first ten rectangular numbers.

Flowchart


START
INITIALIZE
C=1
1
CALCULATE
R=C*(C+1)
PRINT
R
STEP UP
C=C+1
IS
C>10
?
YES
2
NO
1
2
END

Planned Output

```
DISPLAYING RECTANGULAR NUMBERS
(Skip 2 lines.)
2  6  12  20  30  42  56  72  90  110
```

SUMMARY:

1. What is meant by a triangular number?
2. What is the 10th triangular number?
3. What new, important computer concept did we use today?

HOMEWORK:

1. Find the 6th triangular number.
2. Find the 18th triangular number.
3. Find the 20th triangular number.
4. Revise the program we wrote today so that the triangular numbers are displayed horizontally instead of vertically.
5. Which lines in today's program form the counter?
6. Which lines in today's program form the accumulator?

HOMEWORK SOLUTIONS:

1. 21
2. 171
3. 210
4. Change line 40 to PRINT T;
5. Lines 20, 50, 70 and 80. (Lines 40 and 60 can also be included.)
6. Lines 30 and 60.

Lesson: 22

AIM: What is the difference between an arithmetic and geometric sequence?

PERFORMANCE OBJECTIVES: The student will be able to...

1. identify a sequence as arithmetic, geometric, or neither;
2. find the common difference for an arithmetic sequence;
3. find the common ratio for a geometric sequence;
4. find the value of terms in an arithmetic sequence.

VOCABULARY: sequence, arithmetic sequence, geometric sequence, term

MOTIVATION:

After graduation, someone got a job offer with two possible salaries from which to choose.

Plan A: Get a steady $21,400 for the first 10 years.

Plan B: Get an $18,000 salary the first year, but have a yearly increment of $1000 for the next 9 years.

Which plan should he/she take?

DEVELOPMENT:

1. Discuss that the drawing card in Plan A is the higher salary at the beginning. This may be attractive if the money is needed.
2. Discuss the drawing card in Plan B; although the initial pay is lower, the later salaries are better! Develop the sequence of salaries in Plan B...
   18,000, 19,000, 20,000, 21,000, 22,000, 23,000, 24,000, 25,000, 26,000, 27,000.
   Explain that this is a special type of mathematical sequence called an "arithmetic sequence."
3. Write the following definition on the chalkboard.
   An arithmetic sequence (or progression) is a sequence in which the difference between any two successive terms is always the same. This number is called the "common difference", d.

4. In each case, tell whether or not the sequence is arithmetic. If it is, find the value of d.
   a) 5, 10, 15, 20, 25,...
   b) 1, 2, 4, 8, 16,...
   c) 18, 16, 14, 12, 10,...
   d) 1, 1, 1, 1, 1,...
   e) 1, -1, 1, -1, 1,...
   f) 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3,...
   g) 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$,...
   h) 100, 50, 25, $12\frac{1}{2}$,...

   (Note that d can be negative, or even zero. See examples c and d above.)

5. Ask the students to note that in example b above, the rule is to double each number to obtain the successive number. In other words, the ratio between any two successive terms is always the same. Ask, "What is the ratio between any two successive terms in example b?" Write the following definition on the chalkboard. A geometric sequence (or progression) is a sequence in which the ratio between any two successive terms is always the same. This number is called the "common ratio", r.

6. In the examples in 4 above, identify the sequences which are geometric. Find the value of r in those cases. (Note that a sequence can be both arithmetic and geometric. See example d above.)

7. Say to the students that they will be exploring sequences a little more in depth. First, ask them to complete the problems listed in the Applications.

APPLICATIONS:

1. For each sequence, state the rule used to obtain the terms of the sequence. Identify the sequence as arithmetic, geometric or neither and, where possible, find d or r.
   a) 7, 10, 13, 16, 19,...
   b) 1, 1, 2, 3, 5, 8, 13,...
   c) 3, 6, 12, 24,...
   d) 50, 49, 48, 47,...
   e) 10, $9\frac{7}{8}$, $9\frac{3}{4}$, $9\frac{5}{8}$, $9\frac{1}{2}$,...
   f) 1800, 600, 200, $66\frac{2}{3}$, $22\frac{1}{3}$,...
   g) -5, 10, -20, 40, -80,...
   h) -10, -5, 0, 5, 10,...

2. Find the missing terms in each arithmetic sequence.
   a) 10, ___,14, 16, ___,...
   b) 30, 30.5, 40, ___, ___,...
   c) 23, 20, 17, ___, ___,...
   d) ___, ___, ___, 60, 70, 80,...

3. Find the missing terms in each geometric sequence.
   a) ___, 5, 25, ___,...
   b) 3, 12, ___, ___,...
   c) 100, ___, 25, $12\frac{1}{2}$, ___,...
   d) $\frac{1}{4}$, 1, ___, ___,...

SUMMARY:

1. What is the difference between an arithmetic and a geometric sequence?

2. Is the sequence of triangular numbers arithmetic, geometric, or neither? Why?

HOMEWORK:

1. Identify each as an arithmetic sequence, a geometric sequence, or neither.
   a) 120, 122, 124, 126,...
   b) 7, -7, 7, -7, 7,...
   c) 62, 58, 54, 50,...
   d) 6, 7, 9, 12, 16,...
   e) -5, -5, -5, -5,...
   f) 20, 10, 5, $2\frac{1}{2}$,...
   g) 3, 15, 75, 375,...
   h) 1, 4, 9, 16,...
   i) -1600, 400, -100, 25,...

2. Where possible, find the value of d or r in the sequences above.

3. Find the missing terms in the following sequences. (Each sequence is either arithmetic or geometric.)
   a) 10, 15, 20, ___, ___,...
   b) 11, 22, 33, ___, ___,...
   c) 11, -22, 33, ___, ___,...
   d) 5, $5\frac{1}{4}$, $5\frac{1}{2}$, ___, ___,...
   e) ___, 30, ___, 30.4, 30.6,...
   f) ___, 81, ___, 9, 3,...

g) 100, ____, 25, $12\frac{1}{2}$, ____,...

h) 10, -1, .1, ____, ____,...

i) 400, 200, 0, ____, ____,...

HOMEWORK SOLUTIONS:

1. arithmetic: a, c, e, h
   geometric: b, e, f, g, i
   neither: d

2. a) $d = 2$
   b) $r = -1$
   c) $d = -4$
   e) $d = 0$, $r = 1$
   f) $r = \frac{1}{2}$
   g) $r = 5$
   h) $d = 1$
   i) $r = \frac{-1}{4}$

3. a) 25, 30
   b) 44, 55
   c) -44, 55
   d) $5\frac{3}{4}$, 6
   e) 29.8, 30.2
   f) 243, 27
   g) 50, $6\frac{1}{4}$
   h) -.01, .001
   i) -200, -400

Lesson: 23

AIM: How do we find the value of an arithmetic series?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the sum of the terms in an arithmetic sequence using a formula;
2. distinguish between a sequence and a series.

VOCABULARY: arithmetic series, the value of an arithmetic series

MOTIVATION:

Listed below is the salary problem we discussed yesterday. After graduation, someone got a job offer with two possible salaries from which to choose.

Plan A: Get a steady $21,400 for the first 10 years.

Plan B: Get an $18,000 salary the first year, but have a yearly increment of $1000 for the next 9 years.

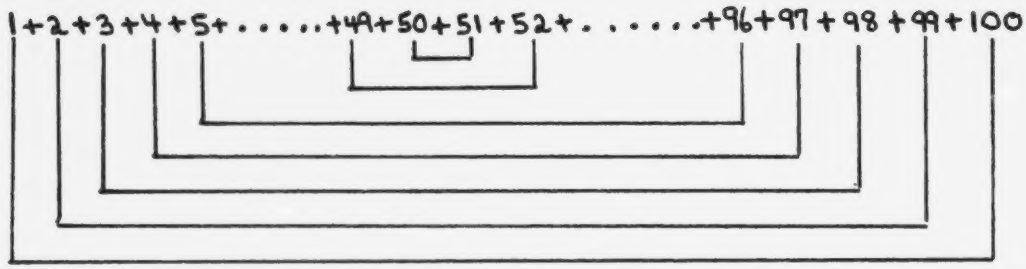
Find the total earnings for ten years under each plan.

DEVELOPMENT:

1. Elicit responses to the motivation. (Plan A: $214,000, Plan B: $225,000) Say to the students that, as in the salary problem, often we are interested in the sum of the numbers in an arithmetic sequence. Write the following definition on the chalkboard.

When we denote the sum of the numbers of an arithmetic sequence, this is called an arithmetic series. This sum is called the value of the series.

2. Say to the students that Karl Frederick Gauss, later to become a very famous mathematician, when he was only ten years old, was asked to find the value of the arithmetic series 1+2+3+4+...+100. He came up with the following scheme which leads to a formula which is still credited to him.



Ask the students to note that each pair adds up to 101. Ask, "How many such pairs are there?" Elicit that there are 50 pairs. Thus, the sum is 101(50) = 5050. (Don't believe it; add in the usual way.) In general, all we need to do is add the first and last terms. (Use f for the value of the first term and l for the value of the last term and n for the number of terms.)

Ask, "How many pairs do we have?" $(\frac{n}{2})$
Therefore, the sum becomes the number of terms divided by two, times the sum of the first and last terms.

3. In symbols, the formula for the value of an arithmetic series is written

$$\underset{\text{value of the sum}}{\underset{\uparrow}{S}} = \frac{\overset{\text{number of terms}}{\overset{\downarrow}{n}}}{2}(\underset{\text{first term}}{\underset{\uparrow}{f}}+\underset{\text{last term}}{\underset{\uparrow}{l}}).$$

4. Ask the students to complete the Applications examples which follow.

APPLICATIONS:

Use the formula to find the value of the following arithmetic series.

1) 18000+19000+20000+...+27000
2) 1+2+3+4+...+100
3) 2+4+6+8+10+12+14+16+18+20
4) 5+8+11+14+17+20+23+26
5) 17+37+57+77+97+117

6) The sum of all the odd counting numbers from 1 through 99.

7) The sum of all the even whole numbers from 2 through 100.

SUMMARY:

1. What is the formula for finding the value of an arithmetic series?
2. What is the advantage in using such a formula rather than just adding the numbers?

HOMEWORK;

In examples 1-7, use the formula $S = \frac{n}{2}(f+l)$ to find the value of each arithmetic series.

1) 3+6+9+12+15+18+21+24
2) 20+25+30+35+40+45+50
3) 100+90+80+70+60+50
4) 1000+999+998+...+1
5) (-5)+(-3)+(-1)+(+1)+(+3)+(+5)+(+7)+(+9)
6) $6+6\frac{1}{4}+6\frac{1}{2}+6\frac{3}{4}+7$
7) The sum of all natural numbers less than 100 that are divisible by 11.
8) What is the difference between a sequence and a series?

HOMEWORK SOLUTIONS:

1) 108
2) 245
3) 450
4) 500,500
5) +16
6) $32\frac{1}{2}$ or 32.5
7) 495
8) A sequence is a list of numbers whereas, a series is an addition example whose addends are the numbers in a sequence. The one number that represents the sum of the arithmetic series is called the value of the series.

Lesson: 24

AIM: How do we find the value of a geometric series?
(Calculators would be useful for this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to...

find the sum of the terms in a geometric sequence using a formula.

VOCABULARY: geometric series, the value of a geometric series

MOTIVATION:

A golf player, feeling overconfident, bet his opponent \$1 on the first hole, double that for the second hole, and so on, doubling the previous amount each time. He quit after losing all eighteen holes! How much money did he lose?

DEVELOPMENT:

1. Ask the students to guess how much money the golf player lost altogether. Ask them to list the amount of money he lost for each hole: 1,2,4,8,16,32,64,128,256,... Ask, "Which type of sequence is this?" (geometric)
2. In order to find his total loss, we must add, 1+2+4+8+... Ask, "Can we use the formula $S = \frac{n}{2}(f+l)$?" Elicit no, since this is not an arithmetic sequence. Say to the students that a formula for the value of a geometric series is needed.
3. In order to find the value of a geometric series, we use the formula

$$\underset{\substack{\uparrow\\ \text{value of}\\ \text{the sum}}}{S} = \frac{fr^{\overset{\text{number of terms}}{n}} - \overset{\text{first term}}{f}}{\underset{\text{common ratio}}{r} - 1} \qquad \text{where } r > 1 \text{ or}$$

$$S = \frac{f - fr^n}{1 - r} \qquad \text{where } r < 1.$$

Ask, "Why couldn't we use either formula when r=1? How would we find the value of the geometric series 3+3+3+3+3?"

Elicit 5(3) = 15 or viewing the above as an arithmetic series, $\frac{5}{2}(3+3) = \frac{5}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{3}{\cancel{6}}}{1} = 15$.

4. Ask, "What is the total loss that we want to find in the motivation problem? This is equivalent to asking what is the value of the geometric series, $\underbrace{1+2+4+8+\ldots}_{18 \text{ terms}}$?

   What is the value of f?...of n?...of r? Which formula should we use in order to determine the value?" Ask the students to substitute into the formula and evaluate as follows.

   $$\frac{fr^n-f}{r-1} = \frac{(1)(2)^{18}-1}{2-1} = \frac{2^{18}-1}{1} = 2^{18}-1 = 262,144-1 = \$262,143$$

   (Note to the teacher: The use of calculators would be helpful here.)

   Ask the students to note that they did not need to know the last term in the series, unlike the problems involving arithmetic series.

5. Ask the students to complete the Applications problems.

APPLICATIONS:

1. Find the value of 5+10+20+40+80 using the appropriate formula.
2. Check example 1 by addition.
3. Find the sum of the first six terms of the geometric series 6+12+24+...using the appropriate formula.
4. Check example 3 by addition.
5. Find the sum 400+100+25+6.25+1.5625 using the appropriate formula.
6. Check example 5 by addition.

SUMMARY:

1. What formula gives the value of a geometric series? What information do you need in order to use it?
2. What formula gives the value of an arithmetic series? What information do you need in order to use it?

HOMEWORK:

In examples 1-5, find the value of each series. Be sure to use the proper formula.

1) 3+6+12+24+48+96

2) 1+2+4+8+16+32+64+128+256+512

3) -1 + 1 -1 +1 -1 +1

4) The sum of the even counting numbers that are less than 1001.

5) 8+40+200+...(7 terms)

6) When do we use the formula $S = \frac{n}{2}(f+l)$?

7) For which type of series is the value of the last term unimportant in getting the sum?

HOMEWORK SOLUTIONS:

1) 189

2) 1023

3) 0

4) 501,000

5) 156,248

6) We use the formula $S = \frac{n}{2}(f+l)$ for finding the value of an arithmetic series.

7) For a geometric series, the value of the last term is unimportant in getting the sum.

Lesson: 25L

AIM: How do we code a program that will find the value of an arithmetic series, without Gauss' help?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to find the value of an arithmetic series;
2. demonstrate his/her understanding of an accumulator and of a counter;
3. enter, run and correct his/her program.

VOCABULARY: accumulator

MOTIVATION:

Find the value of each arithmetic series.

1) 1+2+3+4+5+...+20
2) 1+2+3+4+5+...+60
3) 1+2+3+4+5+...+225

DEVELOPMENT:

1. Say to the students that today they will write a program for the computer to do the examples given in the motivation. Since computers can do about one million calculations per second, the computer does not need to use Gauss' formula.
2. Present the following problem for a computer solution.

   Problem: Have the computer read a positive integer, L. Then have it calculate and display the value of the arithmetic series, 1+2+3+4+...+L. Use the following data list: 20, 60, 225.
3. Ask, "What important computer ideas will be useful here?" Elicit that a counter will be used to count how many numbers have already been added and an accumulator will be used to add on each successive term of the series.

4. Elicit the following analysis from the students.

<u>Analysis</u>

a) Ask, "Will we enter any values in this program?" (Yes, the value of the last term, L. Thus our program will find the sum 1+2+3+...+L.)

b) Ask, "Since we will be employing a counter here, what are the four stages used in a counter?" (INITIALIZE, CALCULATE etc., STEP, CHECK)

c) Ask, "What will be the initial value of our counter, C? (1) How far will we count? (Count until the value in memory cell C passes L.) Will anything else need to be initialized?" (Yes, the value, S, which will be the accumulator, must be initialized at 0.)

5. Develop the following plan with your students.

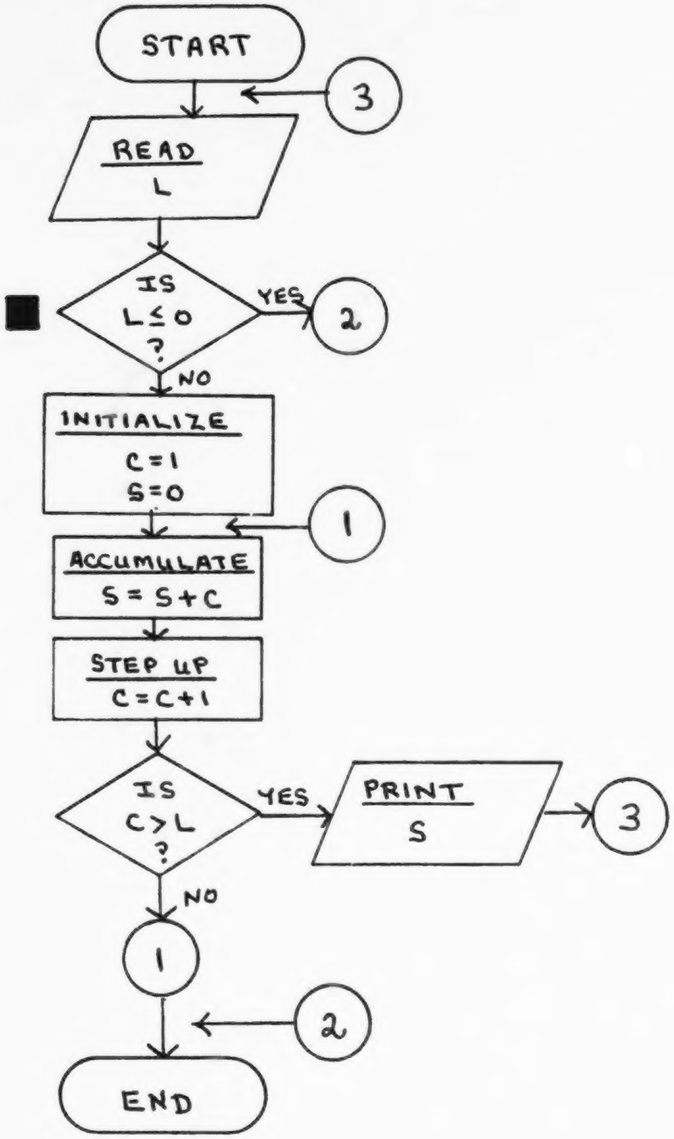
<u>Plan</u>

```
 1)  Read the value of L.
 2)  Check:  Is L < = 0?
          a)  If yes, then go to 10.
          b)  If no, continue.
 3)  Initialize the counter:  C = 1.
 4)  Initialize the accumulator: S = 0.
 5)  Accumulate: S = S+C.
 6)  Step up the count:  C = C+1.
 7)  Check:  Is C > L?
          a)  If yes, print the value of S.  Then go to 1.
          b)  If no, continue.
 8)  Go to 5.
 9)  Data 20, 60, 225, -5
10)  End.
```

6. Develop the following flowchart with the students.

■ Note the use of a flag to signal the computer that there is no more data.

Flowchart


START
3
READ
L
IS
L≤0
?
YES
2
NO
INITIALIZE
C=1
S=0
1
ACCUMULATE
S = S+C
STEP UP
C=C+1
IS
C>L
?
YES
PRINT
S
3
NO
1
2
END

APPLICATIONS:

1. Trace the flowchart using DATA 5, 0.
2. Design a planned output and code a program that will solve the problem discussed in this lesson. (An example of a solution is listed below.)

Planned Output

```
SUM OF THE POSITIVE INTEGERS, 1+2+3+...L
(Skip a line.)
THE SUM OF THE FIRST 20 POSITIVE INTEGERS IS 210
THE SUM OF THE FIRST 60 POSITIVE INTEGERS IS 1830
THE SUM OF THE FIRST 225 POSITIVE INTEGERS IS 25200
(Skip a line.)
BYE BYE!
```

Program

```
10     PRINT "SUM OF THE POSITIVE INTEGERS, 1+2+3+...+L":PRINT
20     READ L
30     IF L < = 0 THEN GOTO 130
40     C = 1
50     S = 0
60     S = S+C
70     C = C+1
80     IF C > L THEN GOTO 100
90     GOTO 60
100    PRINT "THE SUM OF THE FIRST";L;"POSITIVE INTEGERS
IS";S
110    GOTO 20
120    DATA 20, 60, 225, -5
130    PRINT:PRINT "BYE BYE!"
140    END
```

3. RUN your program on the computer.

SUMMARY:

Which computer ideas enable us to write a program to find the value of an arithmetic series?

HOMEWORK:

Select one of the following problems. For the chosen problem, develop a(n) analysis, plan, flowchart, and planned output, and then code the program.

(Give several days for this project. You may wish to hold a laboratory session later on in the week in order to permit the students to run their programs.)

1. Enter a positive integer, L. Have the computer find and display the value of 1+2+3+...+L using Gauss' formula.
2. Enter an even counting number, L. Have the computer calculate and display the value of 2+4+6+8+...+L.
3. Enter an odd counting number, L. Have the computer calculate and display the value of 1+3+5+7+...+L.

*4. Enter a positive integer, L. Have the computer calculate and display the value of $1^2+2^2+3^2+4^2+...L^2$.

*5. Enter an even counting number, L. Have the computer calculate and display the value of the sum of the even squares, $2^2+4^2+6^2+...+L^2$.

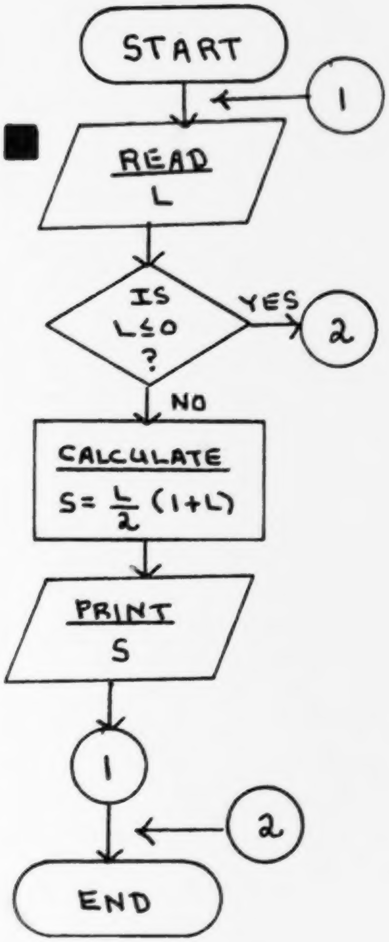
* Are these arithmetic series? Are they geometric series? Why? Why not?

HOMEWORK SOLUTIONS:

(A suggested flowchart, planned output and program for each problem is provided. The required analyses and plans can be gleaned from these.)

1. Flowchart

■ DATA 20, 60, 225, -5


START
1
READ
L
IS
L≤0
?
YES
2
NO
CALCULATE
S= L/2 (1+L)
PRINT
S
1
2
END

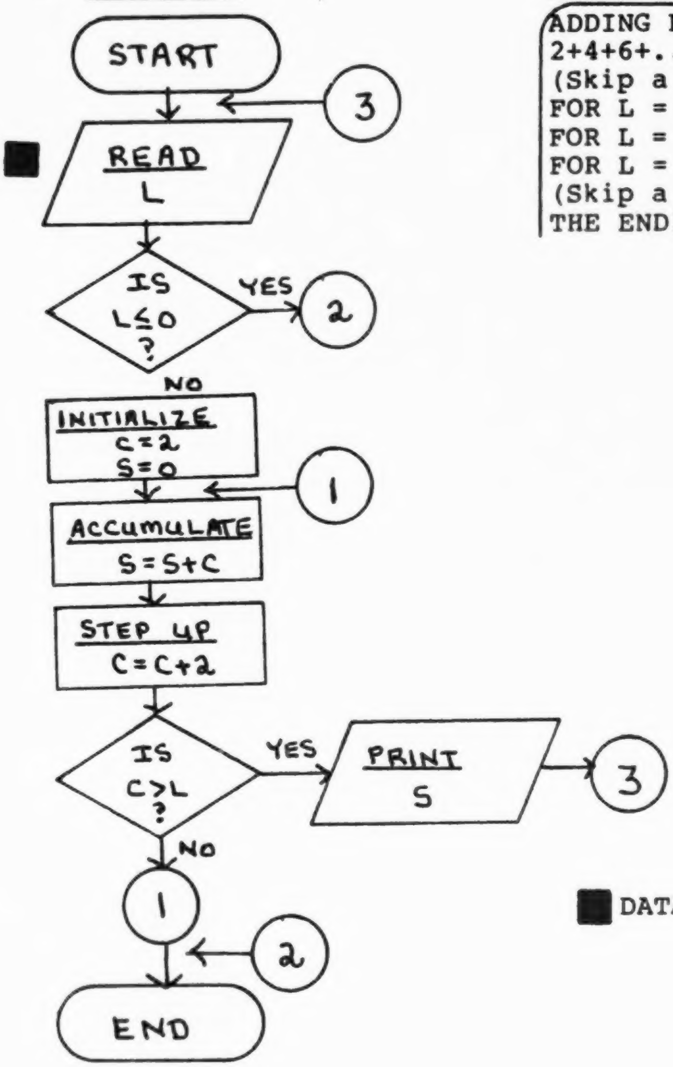
Planned Output

```
ADDING POSITIVE INTEGERS
1+2+3+...+L
(Skip a line.)
THE SUM OF THE 1ST 20 POSITIVE INTEGERS IS 210
THE SUM OF THE 1ST 60 POSITIVE INTEGERS IS 1830
THE SUM OF THE 1ST 225 POSITIVE INTEGERS IS 25200
(Skip a line.)
THE END!
```

Program

```
10  PRINT "ADDING POSITIVE INTEGERS"
20  PRINT "1+2+3+...+L":PRINT
30  READ L
40  IF L < = 0 THEN 90
50  S = L/2*(1+L)
60  PRINT "THE SUM OF THE 1ST";L;"POSITIVE INTEGERS IS";S
70  GOTO 30
80  DATA 20, 60, 225, -5
90  PRINT:PRINT "THE END!"
100 END
```

2. **Flowchart**


START
3
READ
L
IS
L≤0
?
YES
2
NO
INITIALIZE
C=2
S=0
1
ACCUMULATE
S=S+C
STEP UP
C=C+2
IS
C>L
?
YES
PRINT
S
3
NO
1
2
END

Planned Output

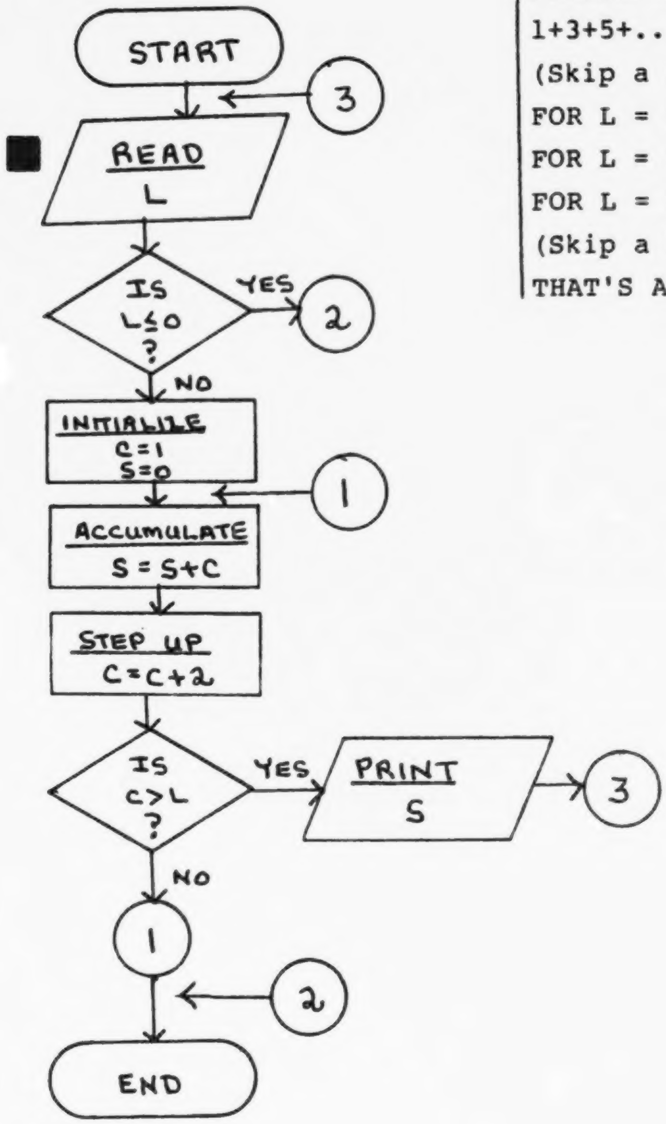
```
ADDING EVEN COUNTING NUMBERS
2+4+6+...+L
(Skip a line.)
FOR L = 20, THE SUM IS 110
FOR L = 60, THE SUM IS 930
FOR L = 224, THE SUM IS 12656
(Skip a line.)
THE END!
```

■ DATA 20, 60, 224, -5

Program

```
10    PRINT "ADDING EVEN COUNTING NUMBERS"
20    PRINT "2+4+6+...+L":PRINT
30    READ L
40    IF L < = 0 THEN 140
50    C = 2
60    S = 0
70    S = S+C
80    C = C+2
90    IF C>L THEN 110
100   GOTO 70
110   PRINT"FOR L =";L;", THE SUM IS";S
120   GOTO 30
130   DATA 20, 60, 224, -5
140   PRINT:PRINT "THE END!"
150   END
```

3. **Flowchart**


START
3
READ
L
IS
L≤0
?
YES
2
NO
INITIALIZE
C=1
S=0
1
ACCUMULATE
S = S+C
STEP UP
C=C+2
IS
C>L
?
YES
PRINT
S
3
NO
1
2
END

Planned Output

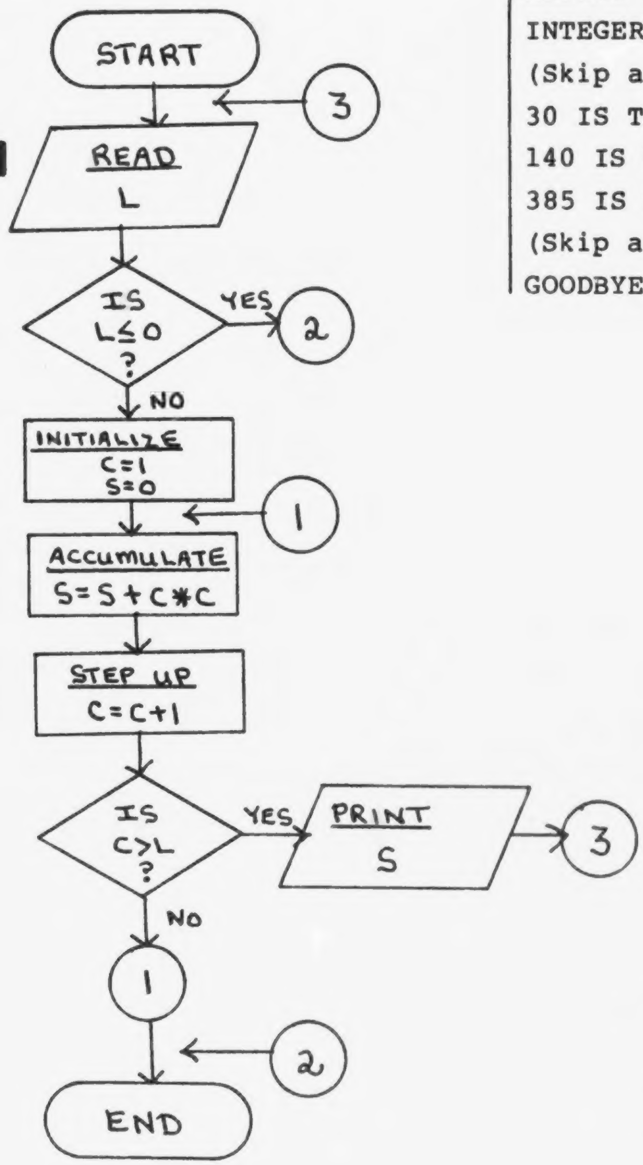
```
ADDING ODD COUNTING NUMBERS
1+3+5+...+L
(Skip a line.)
FOR L = 19, THE SUM IS 100
FOR L = 61, THE SUM IS 961
FOR L = 225, THE SUM IS 12769
(Skip a line.)
THAT'S ALL FOLKS!
```

■ DATA 19, 61, 225, -5

Program

```
10      PRINT "ADDING ODD COUNTING NUMBERS"
20      PRINT "1+3+5+...+L":PRINT
30      READ L
40      IF L < = 0 THEN 140
50      C = 1
60      S = 0
70      S = S+C
80      C = C+2
90      IF C>L THEN 110
100     GOTO 70
110     PRINT"FOR L =";L;", THE SUM IS";S
120     GOTO 30
130     DATA 19, 61, 225, -5
140     PRINT:PRINT "THAT'S ALL FOLKS!"
150     END
```

4. Flowchart


START
3
READ
L
IS
L≤0
?
YES
2
NO
INITIALIZE
C=1
S=0
1
ACCUMULATE
S=S+C*C
STEP UP
C=C+1
IS
C>L
?
YES
PRINT
S
3
NO
1
2
END

Planned Output

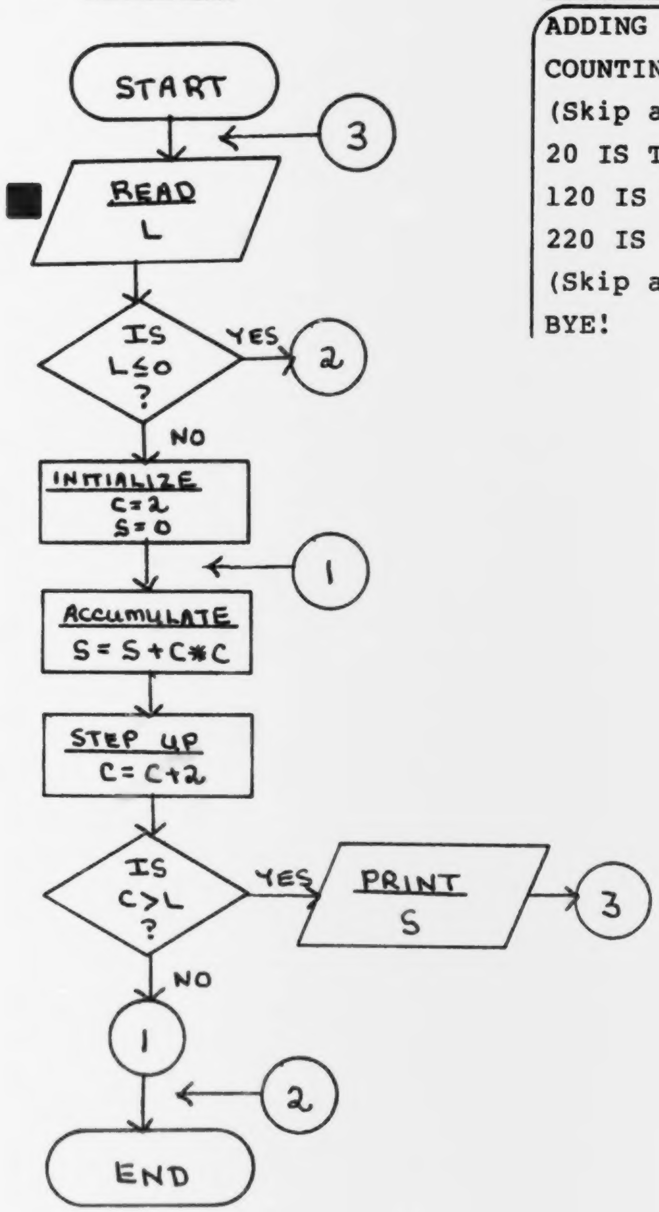
```
ADDING THE SQUARES OF POSITIVE
INTEGERS FROM ONE TO L
(Skip a line.)
30 IS THE SUM OF THE 1ST 4 SQUARES.
140 IS THE SUM OF THE 1ST 7 SQUARES.
385 IS THE SUM OF THE 1ST 10 SQUARES.
(Skip a line.)
GOODBYE!
```

```
DATA 4, 7, 10, -5
```

Program

```
10      PRINT "ADDING THE SQUARES OF POSITIVE"
20      PRINT "INTEGERS FROM ONE TO L":PRINT
30      READ L
40      IF L < = 0 THEN 140
50      C = 1
60      S = 0
70      S = S+C*C
80      C = C+1
90      IF C>L THEN 110
100     GOTO 70
110     PRINT S;"IS THE SUM OF THE 1ST";L;"SQUARES."
120     GOTO 30
130     DATA 4, 7, 10, -5
140     PRINT:PRINT "GOODBYE!"
150     END
```

5. Flowchart


START
3
READ
L
IS
L≤0
?
YES
2
NO
INITIALIZE
C=2
S=0
1
ACCUMULATE
S=S+C*C
STEP UP
C=C+2
IS
C>L
?
YES
PRINT
S
3
NO
1
2
END

Planned Output

```
ADDING THE SQUARES OF EVEN
COUNTING NUMBERS FROM 2 TO L
(Skip a line.)
20 IS THE SUM FOR L  = 4
120 IS THE SUM FOR L = 8
220 IS THE SUM FOR L = 10
(Skip a line.)
BYE!
```

■ DATA 4, 8, 10, -5

Program

```
10     PRINT "ADDING THE SQUARES OF EVEN"
20     PRINT "COUNTING NUMBERS FROM 2 TO L":PRINT
30     READ L
40     IF L < = 0 THEN 140
50     C = 2
60     S = 0
70     S = S+C*C
80     C = C+2
90     IF C>L THEN 110
100    GOTO 70
110    PRINT S;"IS THE SUM FOR L =";L
120    GOTO 30
130    DATA 4, 8, 10, -5
140    PRINT:PRINT "BYE!"
150    END
```

Lesson: 26

AIM: How do we find the factors of numbers?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find all the factors of a given number;
2. develop a routine for the computer to list the factors of a number.

VOCABULARY: factor

MOTIVATION: List all the factors of 18.

DEVELOPMENT:

1. Ask the students to recall the definition of "factors." Ask, "What do we mean by the factors of a number?" Elicit the definition which follows from the students. Definition: The factors of a whole number are those counting numbers that divide the original number with a zero remainder. Demonstrate this idea with 18.

   Since $\begin{array}{r} 18\ R0 \\ 1\overline{)18} \end{array}$, 1 is a factor of 18.

   Since $\begin{array}{r} 9\ R0 \\ 2\overline{)18} \end{array}$, 2 is a factor of 18.

   Since $\begin{array}{r} 6\ R0 \\ 3\overline{)18} \end{array}$, 3 is a factor of 18.

   Since $\begin{array}{r} 4\ R2 \\ 4\overline{)18} \end{array}$, 4 is not a factor of 18...

2. Ask the students to find the factors of each number listed below.

   a) 15 b) 24 c) 10 d) 12 e) 7

3. Say to the students, "Let us figure out how a computer can find the factors of a number."

Problem:

Enter a counting number. Have the computer list all the factors of the number.

4. Develop the following analysis with the students.

Analysis

a) The biggest difficulty will be testing to see if a number leaves a zero remainder when divided into another.

Let N = the number whose factors we desire and
C = a possible factor of N.

b) Let us recall that previously this term (Lesson 17L), we coded a program to determine if a number is divisible by 8. Ask, "Which algorithm did we use in that program to have the computer determine if a number N is divisible by 8?" (If a number N is divisible by 8, then INT(N/8) * 8 = N.) Demonstrate this with N = 16 and N = 10.

c) Ask, "What algorithm do we have the computer use to test to see if N is divisible by 5?"

d) Ask, "What algorithm will we use today to test to see if N is divisible by C?" Elicit INT(N/C) * C = N.

e) Ask, "To get all the factors of N, what numbers shall we try as possible factors in memory cell C?" Elicit 1,2,3,4,...,N.

f) Note that this entails using a counter for C.

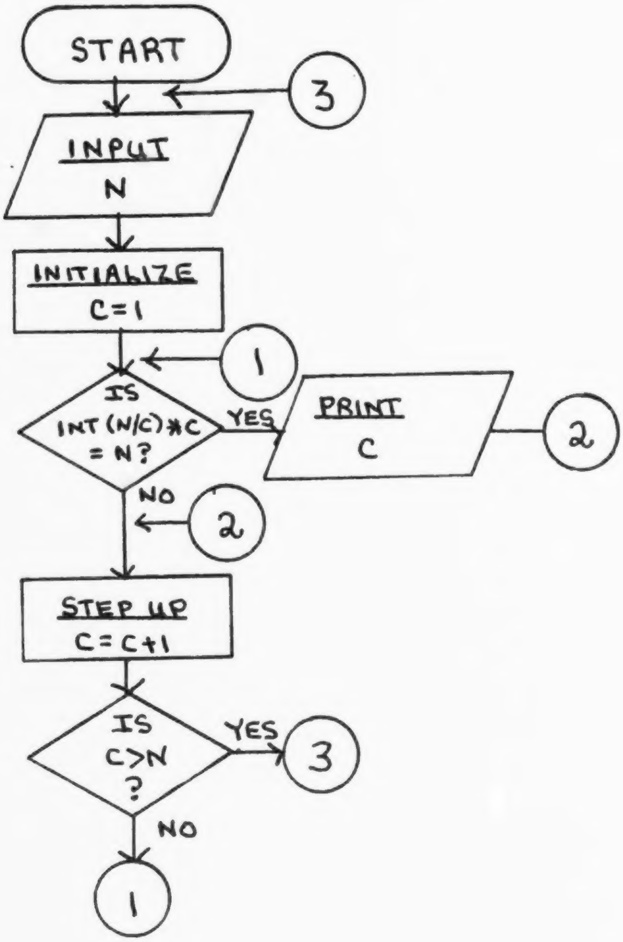
5. Elicit the following plan from the students.

Plan

```
1) Input a value for N, the number whose factors we want
   to find.
2) Initialize:  C = 1, where 1 is the first possible
   factor.
3) Check:  Is INT(N/C) * C = N?
        a) If yes, print C.  Then go to 4.
        b) If no, continue.
4) Step up the value of C to obtain the next possible
   factor:  C = C+1.
5) Check:  Is C > N?
        a) If yes, stop.  Go back to 1 for another
           number for which to obtain factors.
        b) If no, continue.
6) Go to 3 to test this new value of C, which is the
   next possible factor.
```

6. Develop the following flowchart with the students.

Flowchart


START
3
INPUT
N
INITIALIZE
C=1
1
IS
INT (N/C)*C
= N?
YES
PRINT
C
2
NO
2
STEP UP
C=C+1
IS
C>N
?
YES
3
NO
1

7. Ask the students to complete the Applications examples.

APPLICATIONS:

Trace the flowchart developed in this lesson for each of the following values for N.

1) 14
2) 23
3) 9
4) 11
5) 6

SUMMARY:

1. What did the computer idea of "counters" have to do with the mathematical idea of "factors"?
2. Since factors usually come in pairs, most numbers have an even number of factors. Which numbers have an odd number of factors? Give examples.

HOMEWORK

In examples 1-4, find all the factors of the given number.

1) 8
2) 21
3) 64
4) 13
5) Which number has the least factors?
6) Which number has the most factors?

HOMEWORK SOLUTIONS:

1) 1, 2, 4, 8
2) 1, 3, 7, 21
3) 1, 2, 4, 8, 16, 32, 64
4) 1, 13
5) One has only one factor, itself.
6) Zero has the most factors. Every counting number is a factor of zero.

Lesson: 27L

AIM: How do we develop a program to list the factors of a number?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code the program for listing the factors of a number;
2. enter, run and correct his/her program.

VOCABULARY: factor

MOTIVATION:

Trace the flowchart we developed yesterday using the number 36 as data.

DEVELOPMENT:

1. Ask a student to write the flowchart on the chalkboard or use an overhead transparency. Elicit responses to the motivation problem.
2. Say to the students that today they will write a program for listing all the factors of a given number. Let us design an output. Develop one with the students. One possibility follows.

Planned Output

```
FACTORS OF A GIVEN NUMBER
(Skip 2 lines.)
WHAT IS THE GIVEN NUMBER? 36
THE FACTORS ARE:  1 2 3 4 6 9 12 18 36
(Skip 2 lines.)
WHAT IS THE GIVEN NUMBER?
```

3. Using the flowchart (from yesterday) and the planned output in front of them, ask the students to write the program at their seats. One such program follows.

Program

```
10  PRINT "FACTORS OF A GIVEN NUMBER"
20  PRINT:PRINT
30  INPUT "WHAT IS THE GIVEN NUMBER";N
40  PRINT "THE FACTORS ARE: ";
50  C = 1
60  IF INT(N/C) * C = N THEN PRINT C;
70  C = C+1
80  IF C>N THEN GOTO 20
90  GOTO 60
100 END
```

Note that the semicolons in lines 40 and 60 suppress the carriage control and enable printing on the same line. Also, note that the spacing as a result of lines 40 and 60 and the appearance of the question mark as a result of line 30 vary depending on the computer you are using.

4. Ask the students to enter and run this program. Alert students will note that the output differs from the planned output since only one line will be skipped when different values of N are entered. (In line 80, when control is sent to line 20, the carriage is still suppressed from the previous printing (line 60). To rectify this, obtain suggestions from the students. One possible correction is to change line 80 to

```
80 IF C>N THEN PRINT:GOTO 20.)
```

5. Ask the students to complete the Application problem.

APPLICATION:

Revise the program that was written in today's lesson so that in addition to listing all the factors of a given number, it also counts them and displays the number of factors. A suggested planned output follows.

Planned Output

```
HOW MANY FACTORS?
(Skip 2 lines.)
WHAT IS THE GIVEN NUMBER? 36
THE FACTORS ARE: 1 2 3 4 6 9 12 18 36
36 HAS 9 FACTORS.
(Skip 2 lines.)
WHAT IS THE GIVEN NUMBER?
```

SUMMARY:

1. Which computer idea was crucial in solving the problem of listing the factors of a number?
2. How could we make the procedure used here for finding the factors of a number more efficient?

HOMEWORK:

1. Revise the flowchart and program used in today's lesson so that it will produce the planned output below. Read the data using the following list: 36, 24, 18. Be sure that an OUT OF DATA message does not appear.

   Planned Output

   ```
   FACTORS OF A NUMBER
   (Skip a line.)
   THE FACTORS OF 36
   ARE 1 2 3 4 6 9 12 18 36
   (Skip a line.)
   THE FACTORS OF 24
   ARE 1 2 3 4 6 8 12 24
   (Skip a line.)
   THE FACTORS OF 18
   ARE 1 2 3 6 9 18
   (Skip a line.)
   JOB DONE!
   ```

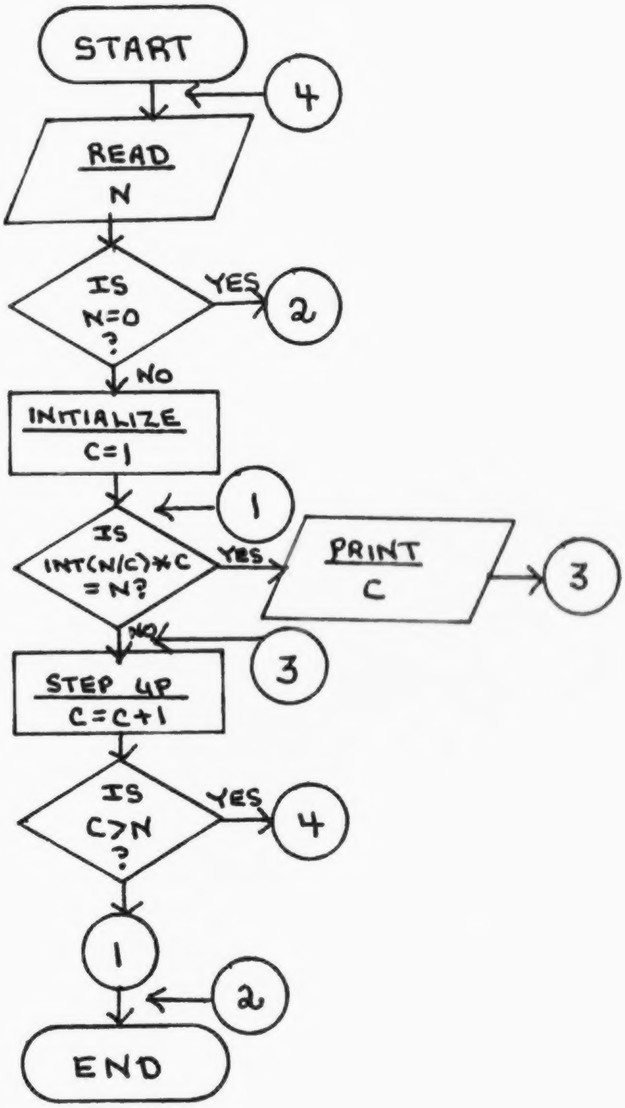
2. Revise the flowchart and program used in today's lesson so that it will produce the planned output which follows. Read the data using the following list: 36, 24, 18. Be sure that an OUT OF DATA message does not appear.

Planned Output

```
FACTORS OF A NUMBER
(Skip 2 lines.)
36 HAS FACTORS 1 2 3 4 6 9 12 18 36
THE SUM OF ITS FACTORS IS 91
(Skip 2 lines.)
24 HAS FACTORS 1 2 3 4 6 8 12 24
THE SUM OF ITS FACTORS IS 60
(Skip 2 lines.)
18 HAS FACTORS 1 2 3 6 9 18
THE SUM OF ITS FACTORS IS 39
```

HOMEWORK SOLUTIONS:

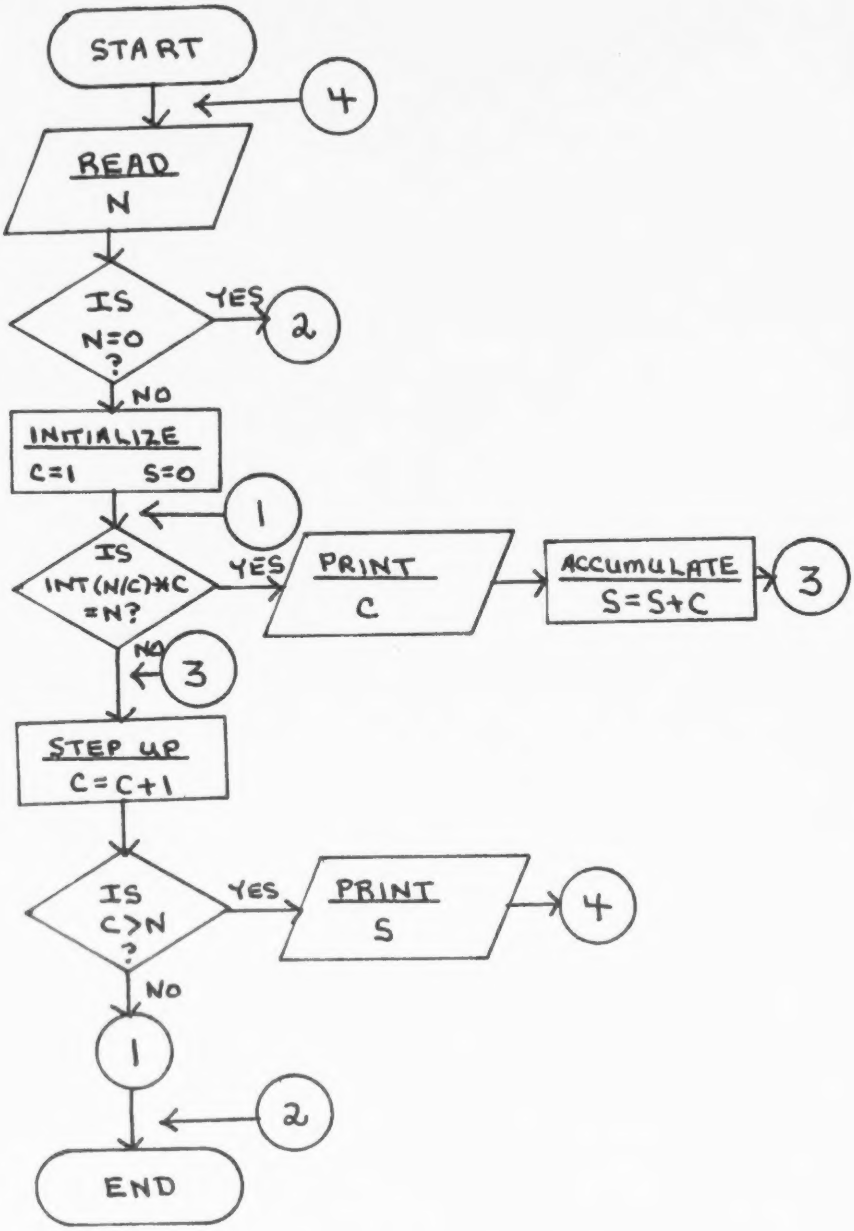
1. Flowchart


START
4
READ
N
IS
N=0
?
YES
2
NO
INITIALIZE
C=1
1
IS
INT(N/C)*C
= N?
YES
PRINT
C
3
NO
3
STEP UP
C=C+1
IS
C>N
?
YES
4
1
2
END

Program

```
10  PRINT "FACTORS OF A NUMBER"
20  PRINT
30  READ N
40  IF N = O THEN GOTO 130
50  PRINT "THE FACTORS OF "; N
60  PRINT "ARE";
70  C = 1
80  IF INT(N/C) * C = N THEN PRINT C;
90  C = C + 1
100 IF C>N THEN PRINT:GOTO 20
110 GOTO 80
120 DATA 36, 24, 18, 0
130 PRINT:PRINT "JOB DONE!"
140 END
```

2. Flowchart


START
4
READ
N
IS
N=0
?
YES
2
NO
INITIALIZE
C=1
S=0
1
IS
INT(N/C)*C
=N?
YES
PRINT
C
ACCUMULATE
S=S+C
3
NO
3
STEP UP
C=C+1
IS
C>N
?
YES
PRINT
S
4
NO
1
2
END

Program

```
10  PRINT "FACTORS OF A NUMBER"
20  PRINT:PRINT
30  READ N
40  IF N = 0 THEN GOTO 130
50  PRINT N; "HAS FACTORS";
60  C = 1 : S = 0
70  IF INT(N/C) * C = N THEN PRINT C; : S = S + C
80  C = C+1
90  IF C > N THEN GOTO 110
100 GOTO 70
110 PRINT:PRINT "THE SUM OF ITS FACTORS IS";S:GOTO 20
120 DATA 36, 24, 18, 0
130 END
```

Lesson: 28L

AIM: How do we use the FOR/NEXT construction in our "factor program"?

PERFORMANCE OBJECTIVES: The student will be able to...

show the output of "simple" programs using FOR/NEXT loops.

VOCABULARY: FOR/NEXT loop

MOTIVATION:

What new computer concept have we learned recently that has often been an excellent aid in our newest programs?

DEVELOPMENT:

1. Elicit the counter concept. Say to the students that since counters come up so often in programming, a special set of instructions called a FOR/NEXT loop was created to simplify using counters.
2. Ask the students to look at the program that they wrote yesterday. Say, "Let's focus in on the counter, lines 50 through 90."
   Write the statements for the counter on the chalkboard as follows.

```
50   C = 1                               (INITIALIZE)
60   IF INT(N/C) * C = N THEN PRINT C;   (CALCULATE,...)
70   C = C+1                             (STEP UP)
80   IF C>N THEN GOTO 20                 (CHECK)
90   GOTO 60
```

3a. Say to the students that the program revised with a FOR/NEXT loop becomes as follows.

```
50   FOR C = 1 TO N
60   IF INT(N/C) * C = N THEN PRINT C;
70   NEXT C
```

Ask the students to note that two less lines are needed. Lines 80 and 90 would be deleted.

3b. Say that in line 50 TWO things happen...C is initialized at 1 and the computer is given the largest value of C (N) that we wish to have it use.

3c. In line 70, NEXT C tells the computer to do TWO things also.

   i) Step up: C = C+1.

   ii) Check this new value of C. If C>N, the computer will pass on to the next line of the program, here line 100; remember, lines 80 and 90 have been deleted. Otherwise, it will go back to the first line after the FOR statement, line 60 here.

4. Ask the students to enter the revised program and verify that the output is indeed the same.

5. Ask the students to complete the Applications problems.

APPLICATIONS:

1a. Show the output for each program.

1b. Verify your results by running the program on the computer.

i)

```
10   FOR A = 1 TO 4
20   PRINT "LOVE"
30   NEXT A
40   END
```

ii)

```
10   FOR B = 2 TO 12 STEP 3
20   PRINT B;
30   NEXT B
40   END
```

iii)

```
10   FOR C = 4 TO 2 STEP -0.5
20   PRINT C;
30   NEXT C
40   END
```

Discuss the fact that if you wish to change the counting variable by any number other than 1, the word and numeral STEP 3, STEP 2, etc., must be used; the 3 or 2 indicates how much to change the variable.

In ii), STEP 3 means that in line 30, the computer steps up B = B+3 until B becomes bigger than 12.

In iii), STEP -0.5 means that in line 30, the computer steps down C = C-.5 until C becomes smaller than 2.

2a. Write a four line program to have the computer count as follows:
5, 10, 15,...100.

2b. RUN your program to verify that it solves the problem.

SUMMARY:

Why do we use FOR/NEXT loops in programs?

HOMEWORK:

Show the output in each case.

1.
```
10   FOR Z = 2 TO 7
20   PRINT Z
30   NEXT Z
40   END
```

2.
```
10   FOR Y = 3 TO 10
20   PRINT Y;
30   NEXT Y
40   END
```

3.
```
10   FOR I = 1 to 3
20   PRINT I; "POTATO"
30   NEXT I
40   END
```

4.
```
10   FOR J=1 TO 10 STEP 2
20   PRINT J;
30   NEXT J
40   END
```

5.
```
10   FOR A = 2 TO 3 STEP 0.2
20   Y = 2 * A
30   PRINT Y;
40   NEXT A
50   END
```

6.
```
10   FOR X = 10 TO 1 STEP -1
20   PRINT X;
30   NEXT X
40   PRINT "BLAST OFF!"
50   END
```

HOMEWORK SOLUTIONS:

1.

Screen	Memory
RUN	Z
2	~~2~~
3	~~3~~
4	~~4~~
5	~~5~~
6	~~6~~
7	~~7~~
	8

2. Screen

```
RUN
3 4 5 6 7 8 9 10
```

Memory

Y

~~3~~

~~4~~

~~5~~

~~6~~

~~7~~

~~8~~

~~9~~

~~10~~

11

3. Screen

```
RUN
1 POTATO
2 POTATO
3 POTATO
```

Memory

I

~~1~~

~~2~~

~~3~~

4

4. Screen

```
RUN
1 3 5 7 9
```

Memory

J

~~1~~

~~3~~

~~5~~

~~7~~

~~9~~

11

5. Screen

```
RUN
4 4.4 4.8 5.2 5.6 6
```

Memory

A	Y
2	2*2 = 4
2.2	2*2.2 = 4.4
2.4	2*2.4 = 4.8
2.6	2*2.6 = 5.2
2.8	2*2.8 = 5.6
3	2*3 = 6
3.2	

6. Screen

```
RUN
10 9 8 7 6 5 4 3 2 1
BLAST OFF!
```

Memory

X

~~10~~

~~9~~

~~8~~

~~7~~

~~6~~

~~5~~

~~4~~

~~3~~

~~2~~

~~1~~

0

Lesson: 29

AIM: What is a prime number?

PERFORMANCE OBJECTIVES: The student will be able to...

1. state whether or not a given number is prime;
2. develop a plan for the computer to determine whether or not a given number is prime;
3. shorten the previous routine for finding the factors of a given number.

VOCABULARY: prime number, composite number

MOTIVATION:

List all the factors of each of the following numbers.

a) 21 b) 5 c) 29 d) 30

DEVELOPMENT:

1. Ask the students to note that the numbers 5 and 29 have exactly two different factors while the other numbers have more than two factors. Write the following two definitions on the chalkboard.

   Definitions:

   i. A prime number is a counting number which has exactly two different whole number factors, itself and one.

   ii. A number with more than two factors is said to be composite.

2. Say to the students that when they are asked to state whether a number is prime or not, they should make a little chart as in the following example.

   Example:

   Which are prime numbers? 14, 23, 81

Number	Factors	Number of Factors	Result
14	1,2,7,14	4	Composite, since there are FOUR factors.
23	1,23	2	Prime, since there are TWO factors.
81	1,3,9,27,81	5	Composite, since there are FIVE factors.

3a. Ask the students to work on the following at their seats, using the above as a model.

Which of the following are prime numbers?

a) 9 b) 6 c) 1 d) 3 e) 31

f) 39 g) 15 h) 48 i) 2 j) 13

3b. Ask, "How many even numbers are prime?" Elicit just one, the number two.

3c. Ask, "Is the number one classified as prime or composite?" Elicit that it is neither prime nor composite since it has only one factor.

4. Ask the students to complete the Applications problem.

5. Say that the next computer project is to enter a number and have the computer display a message telling whether the number is prime, composite or neither.

6. Ask the students to develop the following analysis.

Analysis

a) To tell if a number is prime, we must count the number of factors (NF).

b) If NF > 2, the number is composite.
If NF = 2, the number is prime.
If NF = 1, the number is neither prime nor composite.

c) Since one is the only whole number which is neither prime nor composite, we will just check to see if N = 1.

d) Note that previously in the lesson, we found the factors of 14 to be 1, 2, 7, and 14. The only factor larger than 7 was the original number. Similarly, when we found the factors of 48, the only factor larger than 24 was 48.

In general, we can say that the only factor of N larger than N/2 is N.

e) Since we will test the numbers from 1 to N/2 as possible factors, we will use a FOR/NEXT loop to do this.

f) Ask, "How shall we initialize NF, the number of factors of N?" (NF = 1, since we will not be testing N as a possible factor.)

7. Develop the following plan with your students.

Plan

```
1) Put in a value for a number, N.
2) Is N = 1?
   a) If yes, print NEITHER PRIME NOR COMPOSITE.
      Then go to 1.
   b) If no, continue.
3) Initialize the value in the memory cell that will count
   the number of factors: NF = 1.
4) In order to find other factors, use FOR C = 1 TO N/2.
5) Check: Is INT(N/C) * C = N?
   a) If yes, another factor was found; so step up:
      NF = NF+1. Then go to 6.
   b) If no, continue.
6) NEXT C.
7) Check: Is F = 2?
   a) If yes, print PRIME. Then go to 1.
   b) If no, continue.
8) Print COMPOSITE.
9) Go to 1.
```

APPLICATIONS:

In 1742, Christian Goldbach, a great mathematician, wrote a letter to a friend in which he observed that every even number greater than two could be represented as the sum of two primes. For example,

4 = 2+2 and 8 = 5+3. Show that this is true for 1) 16

2) 18

3) 48.

SUMMARY:

How do you know if a number is prime or composite?

HOMEWORK:

(NOTE: Remind the students to use the divisibility tests considered earilier in the term.)

For each number, determine whether it is a prime number, a composite number or neither.

1. 7
2. 51
3. 17
4. 37
5. 46
6. 1
7. 22
8. 125
9. 1,023,021
10. 83
11. 11
12. 40,048

HOMEWORK SOLUTIONS:

1. prime
2. composite
3. prime
4. prime
5. composite
6. neither
7. composite
8. composite
9. composite
10. prime
11. prime
12. composite

Lesson: 30L

AIM: How do we code a program to determine if a number is prime?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop a flowchart for the prime number program started in Lesson 29;
2. code a program for determining whether a number is prime, composite or neither;
3. enter, run and correct his/her program;
4. use a string variable in a program.

VOCABULARY: string variable

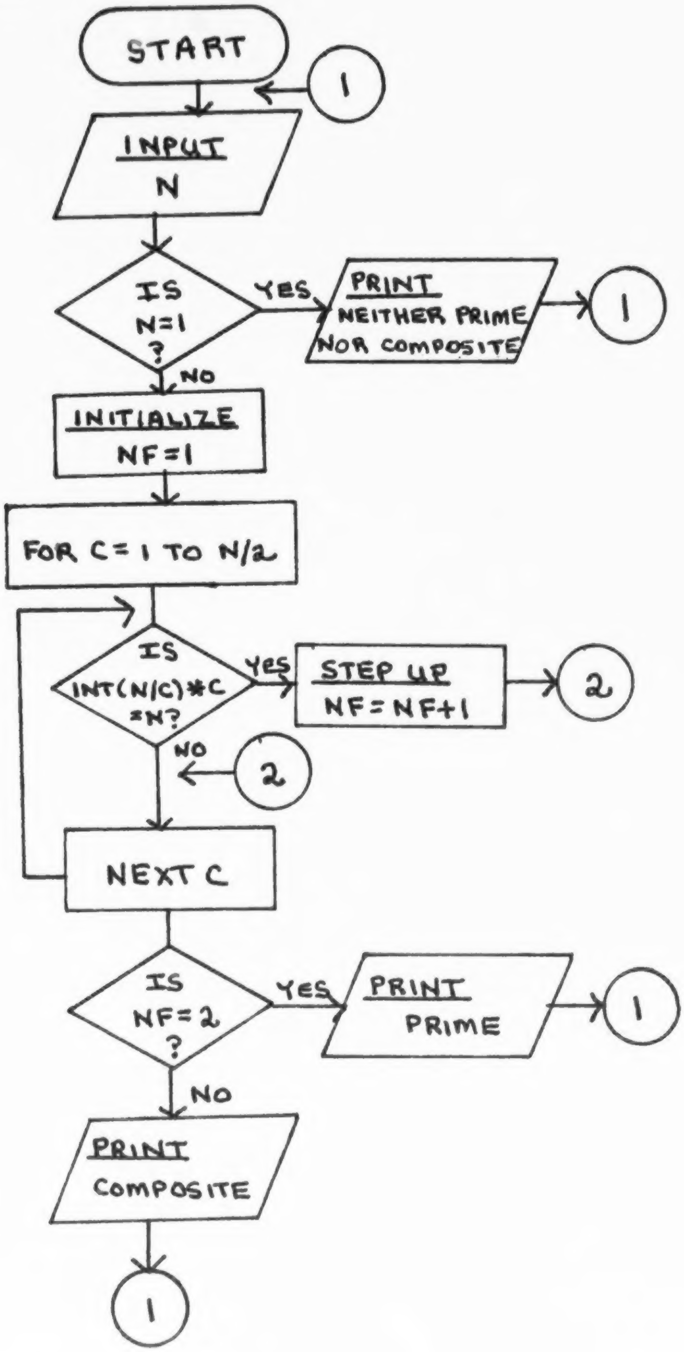
MOTIVATION:

Using the plan written in the last lesson, develop a flowchart for the prime number program.

DEVELOPMENT:

1. Ask a student to place his flowchart on the chalkboard. One possibility follows.

Flowchart


START
1
INPUT
N
IS
N=1
?
YES
PRINT
NEITHER PRIME
NOR COMPOSITE
1
NO
INITIALIZE
NF=1
FOR C=1 TO N/2
IS
INT(N/C)*C
=N?
Yes
STEP UP
NF=NF+1
2
NO
2
NEXT C
IS
NF=2
?
YES
PRINT
PRIME
1
NO
PRINT
COMPOSITE
1

Assist the students in writing the FOR/NEXT loop in flowcharting symbols.

2. Place this or a similar planned output on the chalkboard.

   Planned Output

```
PRIME AND COMPOSITE
(Skip 1 line.)
WHAT IS YOUR NUMBER? 1
1 IS NEITHER PRIME NOR COMPOSITE.
DO YOU WISH TO CONTINUE? YES
(Skip 1 line.)
WHAT IS YOUR NUMBER? 9
9 IS COMPOSITE.
DO YOU WISH TO CONTINUE? YES
(Skip 1 line.)
WHAT IS YOUR NUMBER? 29
29 IS PRIME.
DO YOU WISH TO CONTINUE? NO
(Skip 1 line.)
GOODBYE!
```

3. Ask the students to note that we are giving the user the option of continuing or ending as we did earlier this year. Here, however, we will use an alphabetic input.

4. Develop this program with the students.

   Program

```
10  PRINT "PRIME AND COMPOSITE"
20  PRINT
30  INPUT "WHAT IS YOUR NUMBER"; N
40  IF N = 1 THEN GOTO 150
50  NF = 1
60  FOR C = 1 TO N/2
70  IF INT(N/C) * C = N THEN NF = NF+1
80  NEXT C
90  IF NF = 2 THEN GOTO 170
100 PRINT N; "IS COMPOSITE."
```

```
110 INPUT "DO YOU WISH TO CONTINUE"; A$
120 IF A$ = "YES" THEN GOTO 20
130 PRINT:PRINT "GOODBYE!"
140 GOTO 190
150 PRINT "1 IS NEITHER PRIME NOR COMPOSITE."
160 GOTO 110
170 PRINT N; "IS PRIME."
180 GOTO 110
190 END
```

5. In lines 110 and 120 we used a "string variable." Write the following definition on the chalkboard.
   Definition: A string variable is a variable that names a memory cell that can hold alphabetic data. We name a string variable by placing $ after any legal numeric variable; e.g., A$, PQ$, N1$.
6. Ask the students to enter and run the program. Ask those students who complete the program written here to complete one of the Applications problems.

APPLICATIONS:

1. A proper factor of a number is a factor of the number which is less than the number. Revise the flowchart and program written in today's lesson so that in addition to classifying the number entered as prime, composite or neither, the computer also lists all the proper factors of the number.
2. A perfect number is a number which is equal to the sum of its proper factors; e.g., 6 is a perfect number because 6 = 1+2+3. Revise the flowchart and program written in today's lesson so that rather than classify the number entered as prime, composite or neither, the computer classifies the number as perfect or not perfect.

SUMMARY:

1. What is a string variable?
2. How did we use a string variable today?

HOMEWORK:

1. Rewrite the program we wrote today so that it will produce the output which follows.

```
SEARCHING FOR PRIMES
(Skip 2 lines.)
TELL ME YOUR NUMBER.
? 1
THIS IS NEITHER.
DO YOU WANT TO GO AGAIN? YES
(Skip 1 line.)
TELL ME YOUR NUMBER.
? 9
THIS IS COMPOSITE.
DO YOU WANT TO GO AGAIN? YES
(Skip 1 line.)
TELL ME YOUR NUMBER.
? 17
THIS IS PRIME.
DO YOU WANT TO GO AGAIN? NO
(Skip 1 line.)
IT WAS A PLEASURE!
```

2. Classify each as prime or composite.

   a) 118   b) 64   c) 67   d) 55   e) 107

HOMEWORK SOLUTIONS:

1. Change the following lines in today's program

```
10  PRINT "SEARCHING FOR PRIMES": PRINT
30  PRINT "TELL ME YOUR NUMBER.": INPUT N
100 PRINT "THIS IS COMPOSITE."
110 INPUT "DO YOU WANT TO GO AGAIN"; A$
130 PRINT:PRINT "IT WAS A PLEASURE!"
150 PRINT "THIS IS NEITHER."
170 PRINT "THIS IS PRIME."
```

2. a) composite
   b) composite
   c) prime
   d) composite
   e) prime

Lesson: 31

AIM: How do we determine the square root of a number?

PERFORMANCE OBJECTIVES: The student will be able to...

1. determine the square roots of perfect squares;
2. approximate the square roots of nonperfect squares to the nearest tenth by guessing and squaring;
3. define "irrational number."

VOCABULARY: square root, radical, perfect square, irrational number, radical symbol, principal square root, nonperfect square, opposite, additive inverse
Symbols: $\sqrt{}$, $\approx$

MOTIVATION:

1. What number multiplied by itself gives the product 25?
2. What number multiplied by itself gives the product 81?

DEVELOPMENT:

1. Elicit responses to the motivation questions. Say to the students that 5 is called the square root of 25 because 5x5 = 25 and 9 is called the square root of 81 because 9x9 = 81. Ask the students to note that since negative five times negative five also equals twenty-five, -5 is also called the square root of 25; similarly, -9 is called the square root of 81. Ask, "What is the relationship between 5 and -5?...9 and -9?" Elicit that the two square roots of a number are opposites (or additive inverses). Say to the students that the positive square root of a number is called the principal square root; this will be the one we will be using most in our computer work. Say further that we use the "radical symbol" to indicate the principal square root; e.g., $\sqrt{25} = 5$ and $\sqrt{81} = 9$.

2. Write the following definition on the chalkboard.

   Definition: The square root of a given number is that number which when multiplied by itself, produces the given number. Ask the students to note that the square root of a number is a single number; it is incorrect to say that the square root of 81 is 9x9.

3. Ask the students to evaluate each of the following.

   a) $\sqrt{36}$ b) $\sqrt{1}$ c) $\sqrt{4}$ d) $\sqrt{100}$ e) $\sqrt{144}$

4. Ask the students to recognize that when a number is larger, they may have to list all the possible factor pairs of the number until they reach the one that gives the square root; e.g., for 144 we have the following factors.

   1x144
   2x72
   3x48
   4x36
   6x24
   8x18
   12x12

   Thus, $\sqrt{144} = 12$.

5. Ask the students to evaluate each of the following.

   a) $\sqrt{169}$ b) $\sqrt{196}$ c) $\sqrt{400}$

6. Ask, "How would you find $\sqrt{20}$?" Ask the students to list the factor pairs as follows.

   1x20
   2x10
   4x5

   Say to the students that they may conclude that there is no square root of 20; however, the only conclusion that can be made is that the square root of 20 is NOT a whole number! It is an irrational number, since 20 is not a perfect square. An irrational number is represented by a decimal that does not end. We must therefore approximate the square root. Several methods are possible. Today we will approximate the square root of a nonperfect square to the nearest tenth by guessing and squaring as follows.

Guess #1: Since 20 is between 16 and 25,
$\sqrt{20}$ is between 4 and 5.
So let's guess 4.5.
4.5x4.5 = 20.25, a little too much!

Guess #2: Ask, "How should we change our previous guess so our result won't be too much?" Elicit that we should select a smaller value, 4.4.
Note that 4.4x4.4 = 19.36, a little less than 20!

Ask, "Which of the two guesses came closer to 20?" (As a hint, ask the students to think in terms of money.) Ask the students to conclude that to the nearest tenth, $\sqrt{20} \approx 4.5$.

APPLICATIONS:

Evaluate each of the following to the nearest tenth.

a) $\sqrt{28}$ b) $\sqrt{50}$ c) $\sqrt{90}$ d) $\sqrt{10}$

SUMMARY:

1. What do we mean by the square root of a number?
2. Between which two consecutive integers would you find $\sqrt{75}$?
3. What is the closest perfect square to 15?
4. What is the $\sqrt{.09}$?... $\sqrt{\frac{1}{4}}$?
5. What is the sum of $\sqrt{81}$ and $\sqrt{25}$?

6a. Name a rational number between one and ten.

6b. Name an irrational number between one and ten.

HOMEWORK:

1. Evaluate each.

a) $\sqrt{9}$	d) $\sqrt{121}$
b) $\sqrt{0}$	e) $\sqrt{225}$
c) $\sqrt{49}$	f) $\sqrt{900}$

2. Find each of the following rounded to the nearest tenth.

g) $\sqrt{18}$	i) $\sqrt{70}$
h) $\sqrt{30}$	j) $\sqrt{102}$

3. Bring your calculator tomorrow, if you have one.

HOMEWORK SOLUTIONS:

1a) 3	1f) 30
1b) 0	2g) 4.2
1c) 7	2h) 5.5
1d) 11	2i) 8.4
1e) 15	2j) 10.1

Lesson: 32

AIM: How do we use the Newton-Raphson method for approximating the square root of a number?
(Calculators may be used for this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to...

use the Newton-Raphson method to approximate the square root of a number to the nearest tenth...nearest hundredth.

VOCABULARY: square root, Newton-Raphson method, approximate to the nearest tenth or hundredth

MOTIVATION:

Find $\sqrt{61}$ rounded to the nearest tenth.

DEVELOPMENT:

1. Say to the students that using yesterday's technique, it may have taken them a long time to reach 7.8. After all, they may have squared 7.1, 7.2,..., until they tried 7.8 and 7.9. Another technique makes this process easier; it is called the Newton-Raphson method, named for the two mathematicians credited with the technique.
2. Outline and illustrate the Newton-Raphson method by finding the value of 61 using this method.

Newton-Raphson Method	Example: $\sqrt{61}$
a) Make a good guess.	a) $\sqrt{61}$ is between 7 and 8, but closer to 8 since $7^2 = 49$ and $8^2 = 64$. Guess 7.5.
b) Divide the number by your guess. The answer should have the same number of decimal places as your guess. (Ask the students to use calculators.)	b) $\begin{array}{r} 8.13 \\ 7.5\overline{)61} \end{array} \approx 8.1$

c) Average your guess and the quotient you obtain. This gives a new guess.	c) $\frac{7.5+8.1}{2} = 7.8$
d) Repeat steps b and c with your new guess.	d) $\begin{array}{r} 7.82 \approx 7.8 \\ 7.8\overline{)61} \end{array}$
Keep doing this until the answer does not change.	$\frac{7.8+7.8}{2} = 7.8$

3. At this point, our answer, correct to the nearest tenth, is 7.8. If we want more decimal accuracy, we must allow for more decimal places.

APPLICATIONS:

1. Find $\sqrt{89}$ correct to the nearest hundredth.
   a) Since $9^2 = 81$ and $10^2 = 100$, Guess #1 can be 9.40.
   b) $\begin{array}{r} 9.255 \approx 9.26 \\ 9.40\overline{)87} \end{array}$
   c) $\frac{9.40+9.26}{2} = 9.33$ Guess #2 is 9.33.
   d) Repeat: $\begin{array}{r} 9.324 \approx 9.32 \\ 9.33\overline{)87} \end{array}$;

   $\frac{9.33+9.32}{2} = .9325$ which rounds to 9.33.

2. Use the Newton-Raphson method to find each of the following principal square roots to the nearest tenth.
   a) $\sqrt{55}$
   b) $\sqrt{47}$
   c) $\sqrt{104}$
   d) $\sqrt{165}$

3. Use the Newton-Raphson method to find each to the nearest hundredth.
   a) $\sqrt{62}$
   b) $\sqrt{59}$
   c) $\sqrt{7}$

SUMMARY:

What advantages are there in using the Newton-Raphson method for finding the square root?

HOMEWORK:

1. Use the Newton-Raphson method to find each square root to the nearest tenth.
   a) $\sqrt{11}$ b) $\sqrt{30}$ c) $\sqrt{75}$
2. Use the Newton-Raphson method to find each square root to the nearest hundredth.
   a) $\sqrt{41}$ b) $\sqrt{98}$

HOMEWORK SOLUTIONS:

1. a) 3.3   b) 5.5   c) 8.7

2. a) 6.40   b) 9.90

Lesson: 33

AIM: How do we code a program to find the length of the side of a square, given the area?

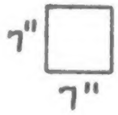
PERFORMANCE OBJECTIVES: The student will be able to...

1. find the length of the side of a square, given the area;
2. write a program for the computer to find the length of a side of a square, given the area;
3. use the SQR(X) function in the above program.

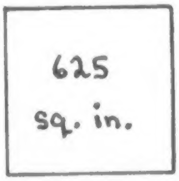
VOCABULARY: argument, square root function, SQR(X)

MOTIVATION:

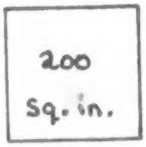
1. Find the area of the square if each side has length 7 inches.



2. Find the length of each side of the square if the area of the square is 625 square inches.



3. Find, rounded to the nearest tenth, the length of each side of the square if the area is 200 square inches.



DEVELOPMENT:

1. Ask, "How do we find the length of the side of a square when we are given the area?" Elicit that to find the length of a side of a square when we are given the area, we must find the principal square root of the area.
2. Say to the students that today they will write a program to solve the following problem.

   Problem: Given the area of a square, have the computer find and display the length of a side of the square.
3. Develop this analysis with the students.

   Analysis: The computer must find the principal square root of the given area. Since finding the square root comes up so often in computer work, a special square root function has been made part of BASIC. The square root function, written SQR(X), where X is called the argument of the function, gives the value of the principal square root of X.

Say to the students that the square root function, SQR(X), actually uses the Newton-Raphson method which they studied yesterday.

Ask the students to note that X cannot be negative.

Example: 10 X=4
20 S=SQR(X) returns a value of 2 for S.

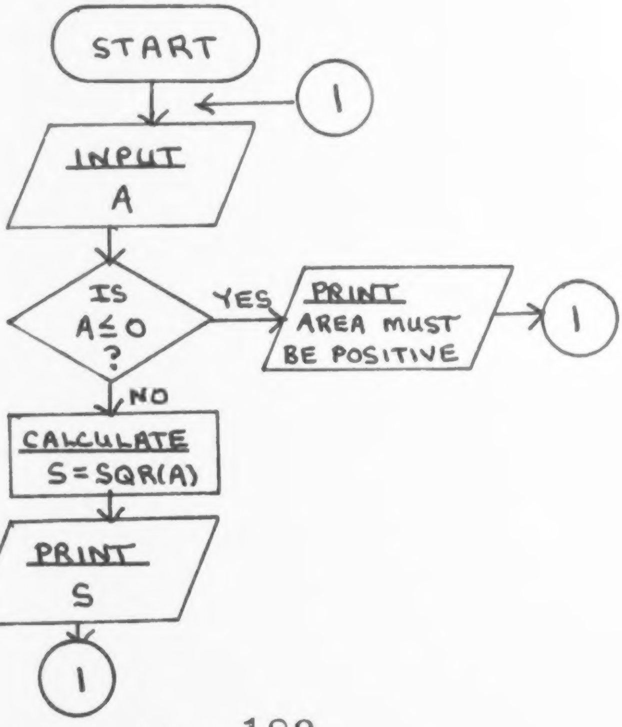
4. Develop this plan with your students.

Plan

1) Enter the area, A.
2) Check: Is $A \leq 0$?
    a) If yes, print AREA MUST BE POSITIVE. Then go to 1.
    b) If no, continue.
3) Calculate the square root: S = SQR(A).
4) Print S.
5) Go to 1.

5. Ask the students to write the flowchart which follows.

Flowchart


START
1
INPUT
A
IS
A≤0
?
YES
PRINT
AREA MUST
BE POSITIVE
1
NO
CALCULATE
S=SQR(A)
PRINT
S
1

6. Ask the students to design a planned output such as the one which follows.

   Planned Output

```
FINDING THE LENGTH OF A SIDE OF A SQUARE
(Skip 1 line.)
WHAT IS THE AREA?
?-64
THE AREA MUST BE POSITIVE.
(Skip 1 line.)
WHAT IS THE AREA?
? 64
THE LENGTH OF A SIDE IS 8
(Skip 1 line.)
WHAT IS THE AREA?
?
```

7. Direct the students to write the program at their seats.

   Program

```
10  PRINT "FINDING THE LENGTH OF A SIDE OF A SQUARE"
20  PRINT:PRINT "WHAT IS THE AREA?"
30  INPUT A
40  IF A < = 0 THEN PRINT "THE AREA MUST BE POSITIVE.":GOTO 20
50  S = SQR(A)
60  PRINT "THE LENGTH OF A SIDE IS "; S
70  GOTO 20
80  END
```

8. Ask the students to complete the Application problem.

APPLICATION:

Revise the flowchart and program written in today's lesson so that given the area, the computer finds the perimeter of the square. Design your own output before coding the program.

SUMMARY:

How does a computer find the square root of a number?

HOMEWORK:

1. Revise the flowchart and the program that we wrote today so that it will produce this output. (Notice that the data is read and no OUT OF DATA message appears.)

   Planned Output

```
THE SIDE OF A SQUARE
(Skip 1 line.)
WHEN THE AREA IS 64, THE SIDE HAS LENGTH 8.
(Skip 1 line.)
WHEN THE AREA IS 100, THE SIDE HAS LENGTH 10.
(Skip 1 line.)
GOODBYE!
```

2. Find the result when executing the two instructions in each.

   a)

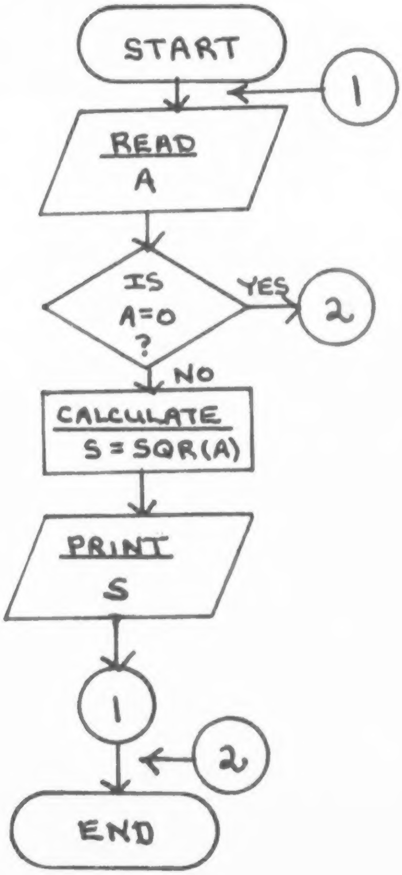
```
10 X = 100
20 Y = SQR(X/4)
```

   b)

```
10 A = 16
20 B = SQR(9*A)
```

HOMEWORK SOLUTIONS:

1. Flowchart


START
1
READ
A
IS
A=0
?
YES
2
NO
CALCULATE
S = SQR(A)
PRINT
S
1
2
END

Program

```
10  PRINT "THE SIDE OF A SQUARE"
20  PRINT
30  READ A
40  IF A = 0 THEN GOTO 90
50  S = SQR(A)
60  PRINT "WHEN THE AREA IS"; A;",";
70  PRINT "THE SIDE HAS LENGTH"; S;"."
80  GOTO 20
90  PRINT:PRINT "GOODBYE!"
100 DATA 64, 100, 0
110 END
```

2. a) Y = 5

   b) B = 12

Lesson: 34

AIM: How do we develop an algorithm to have the computer round a number to the nearest hundredth?

PERFORMANCE OBJECTIVES: The student will be able to...

1. round a number to the nearest whole number, tenth hundredth or thousandth;
2. write an algorithm for rounding to the nearest hundredth;
3. write computer instructions in BASIC to round a number to the nearest hundredth.

VOCABULARY: truncate

MOTIVATION:

If you had only dollar bills and had to pay off a debt right now, how much should you pay if you owed each amount?

a) $15.02
b) $15.18
c) $15.96
d) $15.79
e) $15.45
f) $15.48
g) $15.50

DEVELOPMENT:

1. After eliciting responses to the motivation examples, ask the students to reevaluate the amounts if they have only dollar bills and dimes.
2. Say to the students that the above motivates the need to be able to round off numbers at times. Other examples occur naturally; e.g.,
   - How high, to the nearest foot, is a building...
     a) 56.85 feet tall? b) 72.31 feet tall?
   - How tall are you, to the nearest tenth of an inch, if your height is...
     a) 5 feet 2.34 inches? b) 6 feet 1.57 inches?
3. Ask, "What are the rules for rounding?"

4. Elicit the rules which follow.

RULES: To round off a number...

- a) Separate the digits in the number leaving the correct number of decimal places on the left and the remaining digits on the right;
- b) If the first digit on the right is a digit from zero through four, your answer is the number written on the left.
- c) If the first digit on the right is a digit from five through nine, increase the last digit on the left side by one.

5. Write the following examples on the chalkboard.

a) Round 6102.345 to the nearest tenth.

Solution: 6102.3 45

↑ This number is less than 5.

Answer: 6102.3

b) Round 832.046 to the nearest hundredth.

Solution: 832.04 6

↑ This number is not less than 5. Therefore, we add one to the last digit on the left side.

Answer: 832.05

6. Ask the students to round...

a) 642.31 to the nearest whole number.
b) 83.106 to the nearest tenth.
c) 2.007 to the nearest hundredth.
d) 30.96 to the nearest hundredth.
e) 3.141 to the nearest hundredth.
f) 236.991 to the nearest tenth.
g) 236.991 to the nearest hundredth.

7. Ask, "How can we get the computer to round a number off to the nearest hundredth?"
   a) Say to the students that one method would be to isolate the third place to the right of the decimal point (thousandths' place) and test it to see if it is less than five. This approach would take many lines; let's try another way!
   b) The key to the computer approach will be...
      i) Adding 5 to the thousandths' place. In other words, .005 will be added to the number.
      ii) Afterwards, we will truncate the number to two decimal places. Ask, "What do we mean by the word, 'truncate'?"
   c) Write the following examples on the chalkboard.

Step i) Add .005 to N.	N = 15.123 + .005 15.128	N = 15.129 + .005 15.134
Step ii) Truncate to two decimal places.	15.12	15.13

8. Ask the students to show that the above two steps round each number below to the nearest hundredth.
   a) 16.0741　　c) 1823.0061
   b) 16.0751　　d) 17.02
9. Say that this procedure must be rewritten as computer statements.

TO ROUND A NUMBER N TO THE NEAREST HUNDREDTH:

Steps	Computer Statements
i) Add .005 to N.	R = N + .005
ii) Truncate R to two decimal places.	NN = INT(100*R)/100

10. Ask the students to complete the Applications examples.

APPLICATIONS:

1. Show that the two computer statements discussed in this lesson round off 18.361 and 96.078 properly to the nearest hundredth.

2. How will the two steps differ if we wish to round to the nearest tenth?

SUMMARY:

How will we use the computer to round a number to the nearest hundredth?

HOMEWORK:

1. Round each to the nearest dollar.
    a) $16.04 b) $18.78 c) $154.50
2. Round each number in example 1 to the nearest dime.
3. Round each to the nearest whole number.
    a) 462.37 b) 583.009 c) 962.580
4. Round each to the nearest tenth.
    a) 72.341 b) 82.498 c) 46.009 d) 99.999
5. Round each to the nearest hundredth.
    a) 462.372 b) 582.009 c) 962.580
6. Round each to the nearest thousandth.
    a) 1.04672 b) 3.0581 c) 9.99819
7. Write the computer instructions to round to the nearest...
    a) thousandth. b) integer.

HOMEWORK SOLUTIONS:

1. a) $16 b) $19 c) $155
2. a) $16.00 b) $18.80 c) $154.50
3. a) 462 b) 583 c) 963
4. a) 72.3 b) 82.5 c) 46.0 d) 100.0
5. a) 462.37 b) 582.01 c) 962.58
6. a) 1.047 b) 3.058 c) 9.998

7a)
```
R = N+.0005
NN = INT(1000*R)/1000
```

7b)
```
R = N+.5
NN = INT(1*R)/1  or  NN = INT(R)
```

Lesson: 35L

AIM: How do we revise the program for finding the area of a square so that the computer will round the length of the side to the nearest thousandth?...hundredth?...tenth?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop a flowchart to find the length of the side of a square rounded to the nearest thousandth, hundredth, or tenth when given the area;
2. code a program for the above;
3. enter, run and correct his/her program.

VOCABULARY: SQR(A), INT(A)

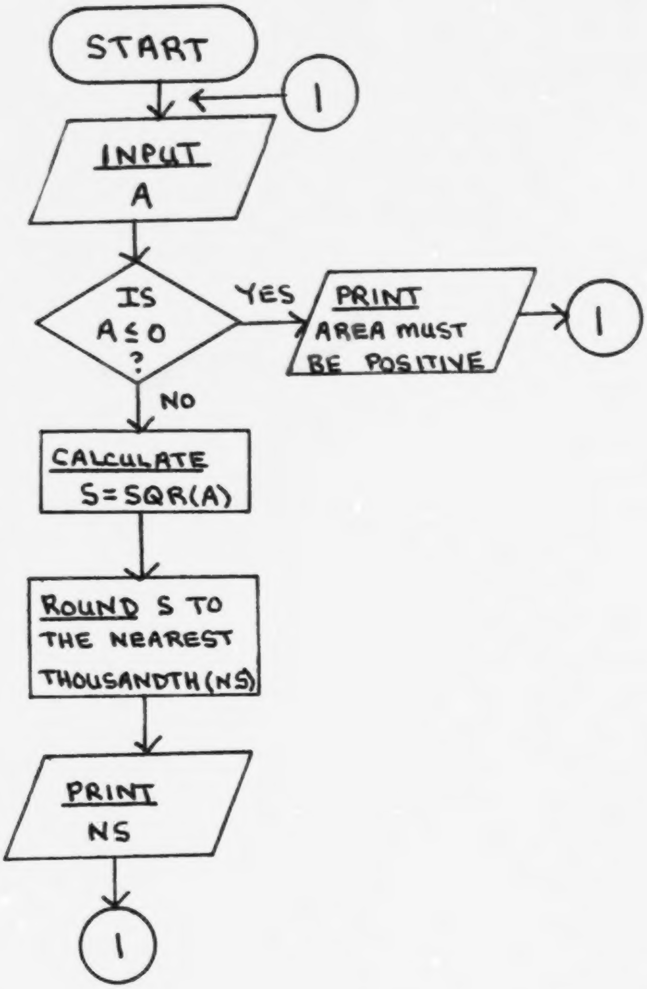
MOTIVATION:

Revise the flowchart from two days ago so that, when given the area, the length of the side of a square is displayed rounded correctly to the nearest thousandth.

DEVELOPMENT:

1. Elicit the following flowchart as a response to the motivation problem.

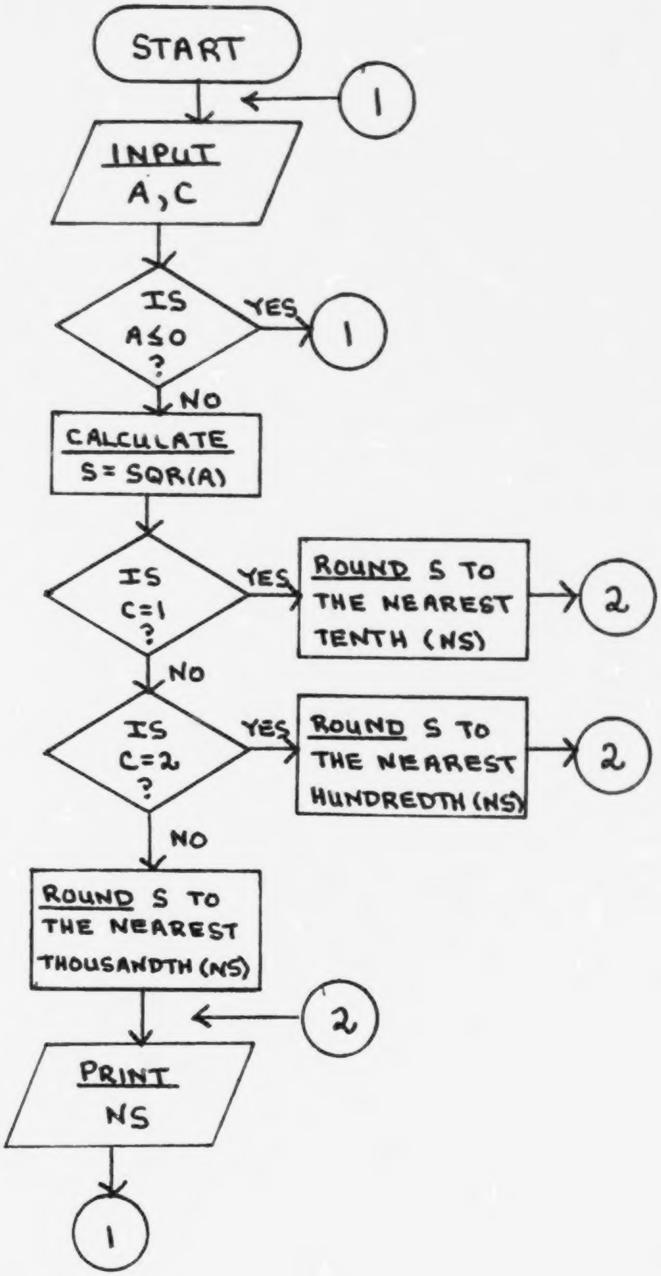
Flowchart


START
1
INPUT
A
IS
A≤0
?
YES
PRINT
AREA MUST
BE POSITIVE
1
NO
CALCULATE
S=SQR(A)
ROUND S TO
THE NEAREST
THOUSANDTH (NS)
PRINT
NS
1

2. Say to the students, "Let's revise this even further.
   Let's allow the user to choose,
   code 3 to round to the nearest thousandth,
   code 2 to round to the nearest hundredth, or
   code 1 to round to the nearest tenth."

3. Ask, "How would we revise the flowchart to permit this choice?" Elicit the changes which follow.

Flowchart


START
1
INPUT
A, C
IS
A≤0
?
YES
1
NO
CALCULATE
S = SQR(A)
IS
C=1
?
YES
ROUND S TO THE NEAREST TENTH (NS)
2
NO
IS
C=2
?
YES
ROUND S TO THE NEAREST HUNDREDTH (NS)
2
NO
ROUND S TO THE NEAREST THOUSANDTH (NS)
2
PRINT
NS
1

4. Ask the students to design a planned output such as the one which follows.

Planned Output

```
THE LENGTH OF A SIDE OF A SQUARE
ROUNDED OFF
(Skip 1 line.)
USE THE FOLLOWING CODES
TO ROUND TO THE NEAREST...
TENTH:      CODE 1
HUNDREDTH:  CODE 2
THOUSANDTH: CODE 3
(Skip 1 line.)
ENTER THE AREA? 55
ENTER YOUR CODE? 3
(Skip 1 line.)
THE SIDES OF THIS SQUARE HAVE LENGTHS
7.416, TO THE NEAREST THOUSANDTH.
(Skip 1 line.)
ENTER THE AREA?-55
ENTER YOUR CODE? 3
THE AREA MUST BE POSITIVE.
(Skip 1 line.)
ENTER THE AREA? 55
ENTER YOUR CODE? 1
(Skip 1 line.)
THE SIDES OF THIS SQUARE HAVE LENGTHS
7.4, TO THE NEAREST TENTH.
(Skip 1 line.)
ENTER THE AREA? 55
ENTER YOUR CODE? 2
(Skip 1 line.)
THE SIDES OF THIS SQUARE HAVE LENGTHS
7.42, TO THE NEAREST HUNDREDTH.
(Skip 1 line.)
ENTER THE AREA?
```

5. Develop the following program with the students.

<u>Program</u>

```
10  PRINT "THE LENGTH OF A SIDE OF A SQUARE": PRINT "ROUNDED
OFF"
20  PRINT
30  PRINT "USE THE FOLLOWING CODES": PRINT "TO ROUND TO THE
NEAREST..."
40  PRINT "TENTH:      CODE 1"
50  PRINT "HUNDREDTH:  CODE 2"
60  PRINT "THOUSANDTH: CODE 3"
70  PRINT
80  INPUT "ENTER THE AREA"; A
90  INPUT "ENTER YOUR CODE"; C
100 IF A<=0 THEN PRINT "THE AREA MUST BE POSITIVE.":GOTO 70
110 S = SQR(A)
120 PRINT: PRINT "THE SIDES OF THIS SQUARE HAVE LENGTHS"
130 IF C = 1 THEN GOTO 190
140 IF C = 2 THEN GOTO 230
150 R = S+.0005
160 NS = INT(1000*R)/1000
170 PRINT NS; ", TO THE NEAREST THOUSANDTH."
180 GOTO 70
190 R = S+.05
200 NS = INT(10*R)/10
210 PRINT NS; ", TO THE NEAREST TENTH."
220 GOTO 70
230 R = S+.005
240 NS = INT(100*R)/100
250 PRINT NS; ", TO THE NEAREST HUNDREDTH."
260 GOTO 70
270 END
```

6. Ask the students to enter and run the program.

7. Ask the students who complete the given program to work on the Application problem.

APPLICATION:

Complete the instructions for the problem which follows.

1. Write an analysis which includes the development of an algorithm.
2. Write a plan for a computer solution to the problem.
3. Develop a flowchart.
4. Design a planned output.
5. Code a program to solve the problem and RUN it on the computer.

Problem:

Have the computer print a chart of the lengths of the sides of all squares whose areas are whole numbers from forty through fifty. The areas should be rounded to the nearest tenth.

SUMMARY:

How did the computer "know" how many places to round the length of the side in the program written in today's lesson?

HOMEWORK:

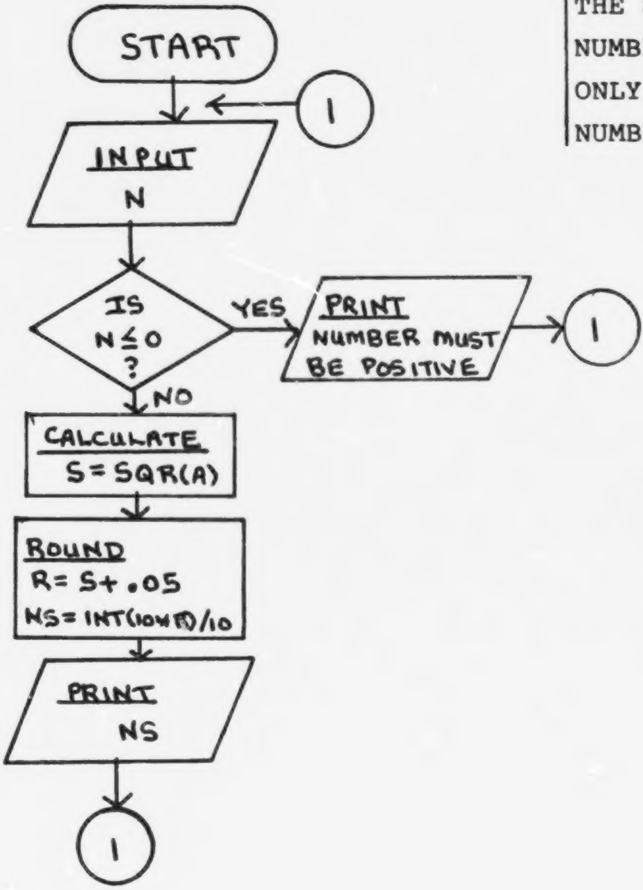
Choose a problem and write out the analysis, plan, flowchart, planned output, and the program. (Allow the students several days to work on this project.)

1. Enter a number. Have the computer print the square root of this number rounded to the nearest tenth.
2. Have the computer print a chart of the square roots of all whole numbers from one through ten. The square roots should be rounded to the nearest hundredth.
3. Enter the area of a square. Have the computer print the perimeter of the square correct to three decimal places.

HOMEWORK SOLUTIONS:

(A suggested flowchart, planned output, and program for each problem is provided. The required analyses and plans can be gleaned from these.)

1. Flowchart


START
1
INPUT
N
IS
N≤0
?
YES
PRINT
NUMBER MUST
BE POSITIVE
1
NO
CALCULATE
S=SQR(A)
ROUND
R= S+.05
NS= INT(10*R)/10
PRINT
NS
1

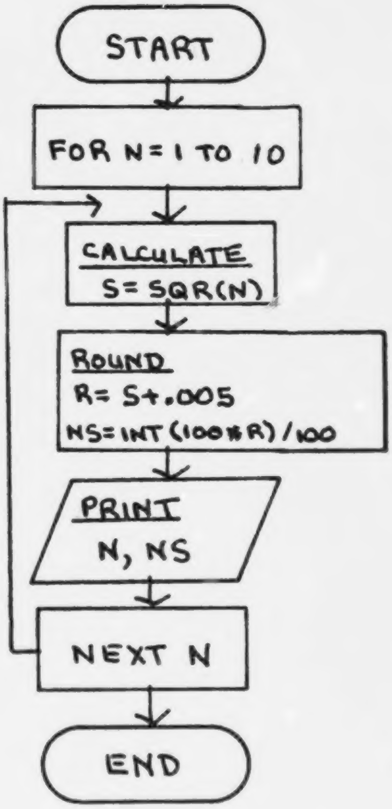
Planned Output

```
ROUNDING A NUMBER
TO THE NEAREST TENTH
(Skip 1 line.)
NUMBER PLEASE? 10
THE SQUARE ROOT OF 10 IS ABOUT 3.2
NUMBER PLEASE?-10
ONLY POSITIVES!
NUMBER PLEASE?
```

Program

```
10    PRINT "ROUNDING A NUMBER"
20    PRINT "TO THE NEAREST TENTH":PRINT
30    INPUT "NUMBER PLEASE"; N
40    IF N < = 0 THEN PRINT "ONLY POSITIVES!":GOTO 30
50    S = SQR(N)
60    R = S+.05
70    NS = INT(R*10)/10
80    PRINT "THE SQUARE ROOT OF";N;"IS ABOUT";NS
90    GOTO 30
100   END
```

2. Flowchart


START
FOR N=1 TO 10
CALCULATE
S=SQR(N)
ROUND
R= S+.005
NS=INT(100*R)/100
PRINT
N, NS
NEXT N
END

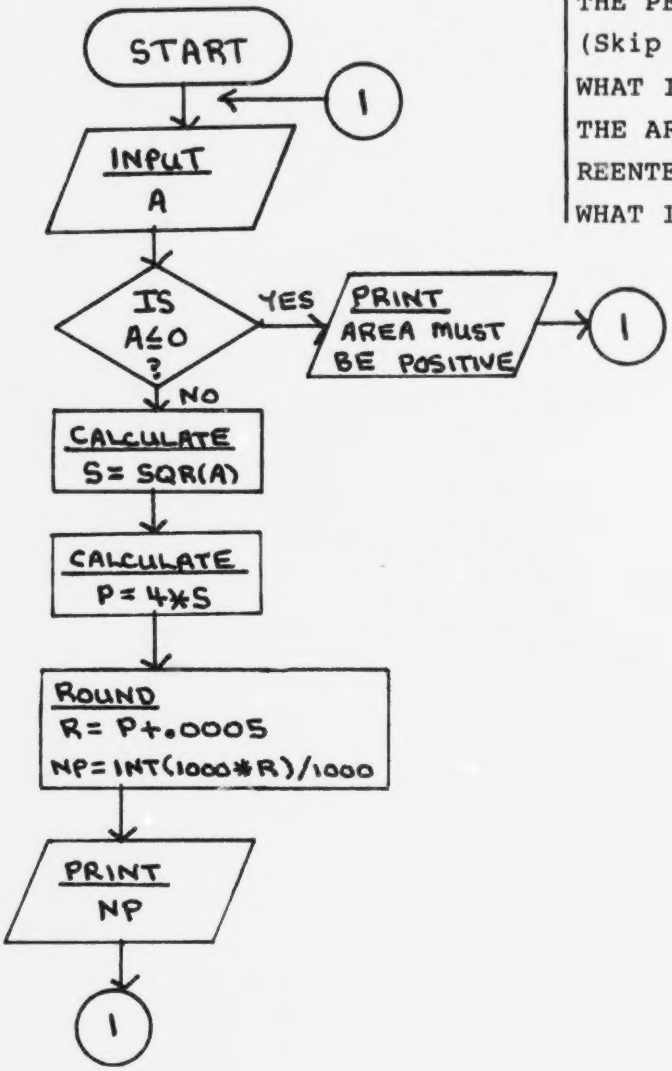
Planned Output

```
SQUARE ROOTS ROUNDED TO THE
NEAREST HUNDREDTH
NUMBERS              SQUARE ROOTS
1                    1
2                    1.41
3                    1.73
4                    2
5                    2.24
6                    2.45
7                    2.65
8                    2.83
9                    3
10                   3.16
```

Program

```
10    PRINT "SQUARE ROOTS ROUNDED TO THE"
20    PRINT "NEAREST HUNDREDTH"
30    PRINT "NUMBERS" , "SQUARE ROOTS"
40    FOR N = 1 TO 10
50    S = SQR(N)
60    R = S+.005:NS = INT(100*R)/100
70    PRINT N, NS
80    NEXT N
90    END
```

3. Flowchart


START
1
INPUT
A
IS
A≤0
?
YES
PRINT
AREA MUST
BE POSITIVE
1
NO
CALCULATE
S= SQR(A)
CALCULATE
P= 4*S
ROUND
R= P+.0005
NP= INT(1000*R)/1000
PRINT
NP
1

Planned Output

```
FINDING THE PERIMETER OF A SQUARE
(Skip 2 lines.)
WHAT IS THE AREA? 55
THE PERIMETER IS ABOUT 29.665
(Skip 1 line.)
WHAT IS THE AREA?-55
THE AREA MUST BE POSITIVE.
REENTER, PLEASE.
WHAT IS THE AREA?
```

Program

```
10     PRINT "FINDING THE PERIMETER OF A SQUARE":PRINT
20     PRINT
30     INPUT "WHAT IS THE AREA"; A
40     IF A < = 0 THEN GOTO 80
50     S = SQR(A):P = 4*S
60     R = P+.0005:NP = INT(1000*R)/1000
70     PRINT "THE PERIMETER IS ABOUT";NP:GOTO 20
80     PRINT "THE AREA MUST BE POSITIVE."
90     PRINT "REENTER, PLEASE."
100    GOTO 30
110    END
```

Lesson: 36

AIM: How do we find the circumference and area of a circle?

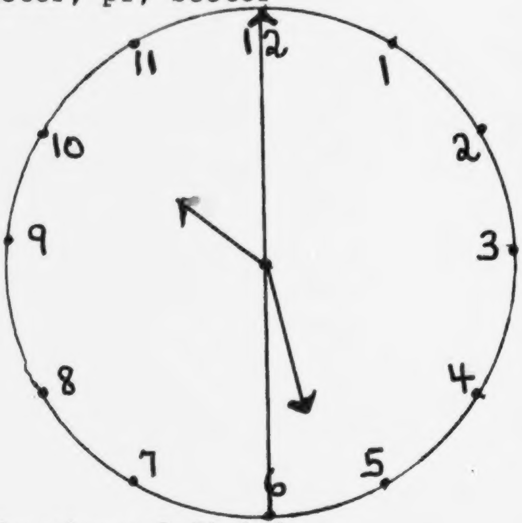
PERFORMANCE OBJECTIVES: The student will be able to...

1. find the circumference of a circle when given the length of its radius or diameter;
2. find the area of a circle when given the length of its radius or diameter;
3. round off his/her answer to the nearest tenth or hundredth.

(Note: You may wish to permit the students to use calculators for the computations in this lesson.)

VOCABULARY: circumference, radius, diameter, pi, sector
Symbols: π

MOTIVATION:



The second hand of this clock touches the outer rim and goes through the center of the circle. If the length of the second hand is 1 foot long, how far will the tip of its arrow go in one complete minute?

DEVELOPMENT:

1. The motivation problem leads to the three definitions which follow.
   a) The distance around a circle is called the circumference. Ask the students to note that this is a special word that is reserved for circles. It means the same thing that perimeter means for polygons.
   b) A diameter is a line segment that connects two points of the circle and passes through the center.
   c) A radius of a circle is a line segment that connects one point of the circle with its center.

Ask, "How does the length of a radius of a particular circle compare with the length of a diameter?" Elicit that the length of a radius is one-half of the length of a diameter, or $r = \frac{1}{2}d$.

2. Explain that for many years, the ancients thought that the circumference of a circle was always three times the length of a diameter ($C = 3D$). In fact, this was even mentioned in the Bible. Ask, "Using this idea, what would be the circumference of the clock in our first example?" Elicit 3 feet. Years later, it was found that $C = D \cdot 3$ or $C = 3D$ was a little inaccurate. A more accurate formula would have been $C = D \cdot 3.14159265358979323846264338...$ The long number written above is an infinite nonrepeating decimal. Since the Greeks first studied it, they used the Greek letter for "p" or π (pi) to stand for the horrible looking number above. Thus, $C = \pi d$ where π is usually approximated by 3.14.

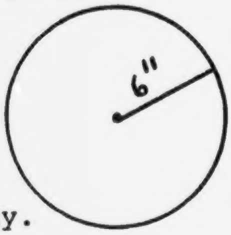
3. Ask the students to complete Applications problems 1, 2 and 3. Be sure to explain that circumference has linear units. Caution them to be careful with the rounding.

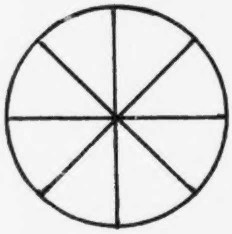
4. Say to the students that in Applications example 3, the radius is given. This can be handled in two ways. Either we can double the length of the radius and use the formula presented earlier or we can alter that formula as follows. Since d has the same value as 2r,

$$C = \pi d$$
$$C = \pi(2r)$$
$$C = 2\pi r.$$

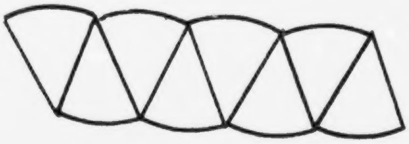
5. Ask the students to complete Applications example 4.

6. Ask, "If the radius of this pizza has a length of 6 inches, what is its area?" At this point, you may wish to develop the formula for the area of a circle in this way.





a) Take a cut-out circle with several diameters drawn, creating equal parts (sectors). Here, 8 equal parts were chosen. You may wish to use more or less, accordingly.



b) Cut out the pieces of "pizza" and arrange them as shown in the diagram at the right.

c) Ask the students to note that this figure is "close" to being a parallelogram. Had we used more "pizza slices" (sectors), the diagram would approximate a parallelogram even more. Ask, "What is the formula for the area of a parallelogram?" Ask the students to recall A = bh. The length of the base is $\frac{1}{2}$ of the circumference = $\frac{1}{2}(2\pi r) = \pi r$. The height is approximately r. (This would be illustrated better with a pre-cut diagram using more sectors.) Thus, $A = (\pi r)r$ or $A = \pi r^2$.

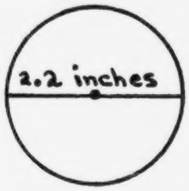
7. Ask "What is the area of the pizza mentioned above if we use $\pi \approx 3.14$?" Ask the students to round the area to the nearest inch. Elicit 113 square inches. Ask the students to recall that the area is always measured in square units.

8. Ask the students to complete Applications problems 5,6 and 7.

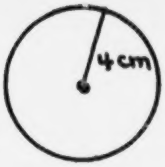
APPLICATIONS:

1. Use $\pi \approx 3.14$ to find the circumference of the clock in the first problem we considered today. Round your answer to the nearest tenth.

2. If the diameter of this circle is 2.2 inches, use $\pi \approx 3.142$ to approximate the circumference of the circle to the nearest tenth of an inch.



3. A spoke of this wheel has length 4 cm. Use $\pi \approx 3.14$ to approximate the circumference to the nearest cm.



4. The radius of a circle is 2 inches. Use $\pi \approx 3.14$ to approximate the circumference to the nearest tenth.
5. Find the area of the circle in 3 above, rounded to the nearest hundredth.
6. Find the area of the circle in 4 above, rounded to the nearest tenth.
7. Find the area of the circle in 2 above, rounded to the nearest tenth.
   (Note that the students should calculate the length of the radius first. Do not develop a formula for the area of a circle in terms of the diameter.)

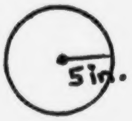
SUMMARY:

1. How do we find the circumference and area of a circle?
2. If the length of the diameter of a circle is doubled, what happens to the circumference?
3. If the length of the radius of a circle is tripled, what happens to the area?

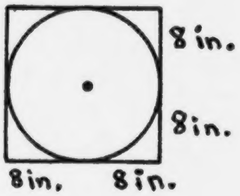
HOMEWORK:

For each problem, find the circumference of the circle using $\pi \approx 3.142$ and round off the answer to the nearest hundredth.

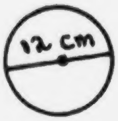
a)



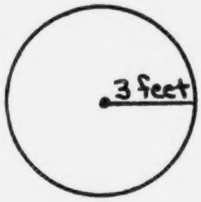
c)



b)



d)



For each problem, find the area using $\pi \approx 3.1416$ and round off the answer to the nearest tenth.

e) Example a) above.
f) Example b) above.
g) Example c) above.
h) Example d) above.

HOMEWORK SOLUTIONS:

a) 31.42 inches
b) 37.70 centimeters
c) 50.27 inches
d) 18.85 feet

e) 78.5 square inches
f) 113.1 square centimeters
g) 201.1 square inches
h) 28.3 square feet

Lesson: 37L

AIM: How do we write a program to have the computer find the circumference of a circle rounded to the nearest hundredth?

PERFORMANCE OBJECTIVES: The student will be able to...

1. analyze, plan (flowchart) and code a program to have the computer find the circumference of a circle to the nearest hundredth;
2. enter, run and correct his/her program.

MOTIVATION:

Let us put some of our recent ideas together into one computer program.

Problem: Enter the length of the radius of a circle. Have the computer find and display the circumference of the circle rounded to the nearest hundredth using $\pi = 3.1416$.

DEVELOPMENT:

1. Elicit the following analysis.

   Analysis

   a) We will use $C = 2\pi R$ to find the circumference. Since the computer does not have a "π" button, we modify this to C = 2*3.1416*R.

   b) We round C using C1=C+.005 and NC=INT(100*C1)/100.

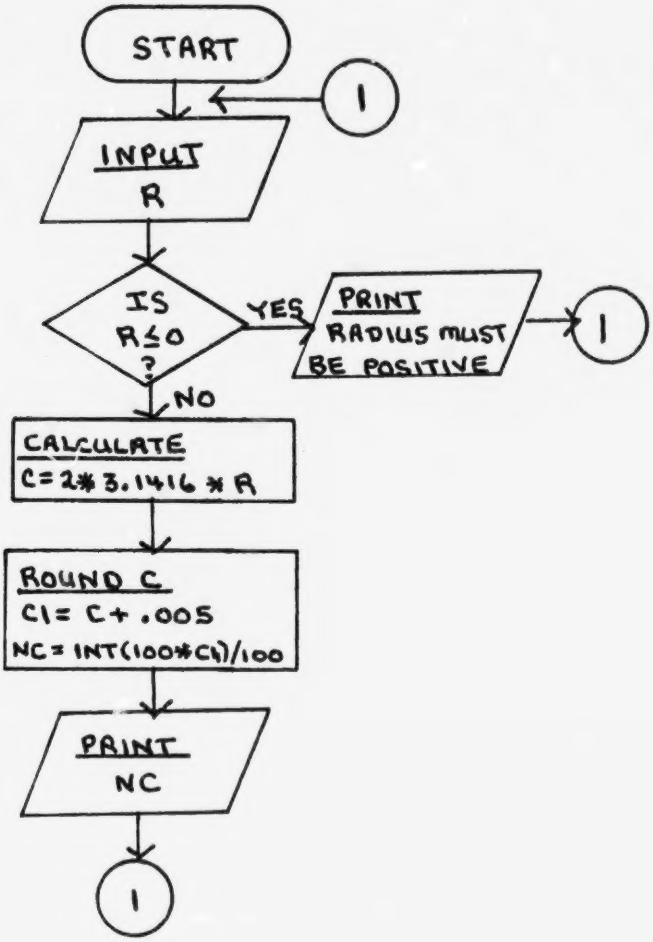
2. Elicit the following plan.

   Plan

   1) Put in a value for the length of the radius, R.
   2) Check: Is $R \leq 0$?
      a) If yes, display an appropriate message and go back to 1.
      b) If no, continue.
   3) Calculate the circumference: C=2*3.1416*R.
   4) Round C to the nearest hundredth:
      C1=C+.005
      NC=INT(100*C1)/100.
   5) Print NC.
   6) Go back to 1.

3. Ask the students to develop the flowchart which follows.

Flowchart


START
INPUT R
IS R≤0 ?
YES
PRINT RADIUS MUST BE POSITIVE
1
NO
CALCULATE C=2*3.1416*R
ROUND C C1= C+.005 NC= INT(100*C1)/100
PRINT NC
1

4. Ask each student to design an output, code the program, enter and run it. Circulate to help those youngsters having difficulties and to give support to the others.

One possible program follows.

Program

```
10  PRINT "FINDING THE CIRCUMFERENCE"
20  PRINT
30  INPUT "TELL ME YOUR NAME"; N$
40  PRINT
50  PRINT "READY ";N$;", TELL ME THE LENGTH OF THE RADIUS."
60  INPUT R
70  IF R<=0 THEN PRINT "BAD DATA, ";N$;".":GOTO 50
80  C = 2*3.1416*R
90  C1 = C + .005: NC = INT(100*C1)/100
100 PRINT N$;", YOUR CIRCUMFERENCE IS";NC;", ROUNDED"
110 PRINT "TO THE NEAREST HUNDREDTH."
120 INPUT "DO YOU HAVE MORE DATA"; A$
130 IF A$ = "YES" THEN GOTO 40
140 PRINT "AU REVOIR!"
150 END
```

5. Ask the students who complete the program to begin the Application problem.

APPLICATION:

Revise today's flowchart and program so that in addition to finding the circumference, the computer also finds the area of the circle.

SUMMARY:

1. What device did we use to get the computer to use the number π?
2. How does a computer find the circumference of a circle when it is given the length of the radius?

HOMEWORK:

Revise today's program so that the length of the diameter is entered and the computer finds and displays the area rounded to the nearest hundredth.

HOMEWORK SOLUTIONS:

Make the following changes:

line 10 Change CIRCUMFERENCE TO AREA.

line 50 Change RADIUS to DIAMETER.

line 60 INPUT D

line 70 Change R<=0 to D<=0.

line 75 R = D/2

line 80 A=3.1416*R*R

line 90 A1=A+.005: NA=INT(100*A1)/100

line 100 Change CIRCUMFERENCE to AREA and NC to NA.

Lesson: 38L

AIM: How do we code programs to determine the areas of composite figures?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the area of a composite figure;
2. write the analysis, plan and program to find the area of a composite figure.

VOCABULARY: surmount, Norman window, inscribed, concentric circles, annular rings

MOTIVATION:

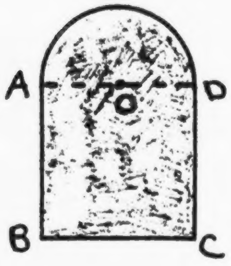
Divide the class into three (or six) groups. Each group will be assigned a different area problem on a rexographed sheet. After finding the area, the group will be given a similar computer project to:

a) analyze
b) plan a program
c) write a flowchart
d) design an output
e) code a program

(The tasks may be broken up among the group members.)

DEVELOPMENT:

GROUP A

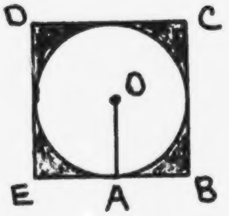


This is called a "Norman" window. It is formed by surmounting a semicircle upon a square.

1. Find, to the nearest tenth, the area of this window (shaded) if AD = 10 inches. Use π = 3.14.
   (Hint: Be careful to find the length of the radius first; then find the area of the semicircle and the square. (178.5 sq. in.)

2. **Computer Project**: Enter the length of $\overline{AD}$. The computer will find the area of the Normam window rounded to the nearest hundredth using $\pi = 3.14159$.

GROUP B

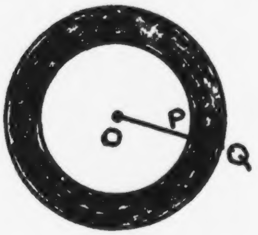


1. The circle is "inscribed" in the square. If the length of the radius $\overline{OA} = 5$, find
   a) the length of a side of the square;
   b) the area of the square;
   c) the area of the circle rounded to the nearest tenth, using $\pi=3.14$;
   d) the shaded area using the previous results.
   (a) 10 b) 100 c) 78.5 d) 21.5)

2. **Computer Project**: Enter the length of the radius of a circle. Have the computer display
   a) the length of a side of the square;
   b) the area of the square;
   c) the area of the circle rounded to the nearest hundredth, using $\pi = 3.14159$;
   d) the shaded area.

GROUP C

1. The two circles have the same center. They are called "concentric circles." The shaded area between the circles is called an "annular ring." (This term comes from biology since it reminds us of the annual rings on trees.) Use $\pi = 3.14$ to solve the following problems.



a) If the length of radius $\overline{OP} = 5$, find the area of the inner circle to the nearest tenth. (78.5)

b) Find the area of the outer circle rounded to the nearest tenth, if the length of radius $\overline{OQ} = 7$. (153.9)

c) Find the area of the annular ring. (75.4)

2. Computer Project: Enter the length of radius $\overline{OP}$ and radius $\overline{OQ}$. Have the computer use $\pi = 3.14159$ to find each to the nearest hundredth:
   a) the area of the inner circle;
   b) the area of the outer circle;
   c) the area of the annular ring.

APPLICATION:

Ask the students to enter, run, and edit the program where necessary. Circulate around the computer laboratory assisting the groups as needed.

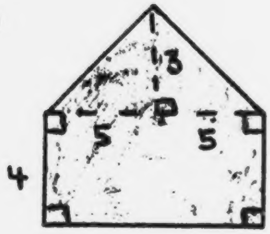
SUMMARY:

Ask one representative from each group to summarize the solution to the numerical problem. Ask another to demonstrate the program. Ask a third to explain the analysis.

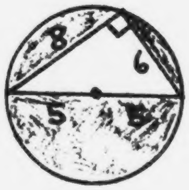
HOMEWORK:

Find each shaded area rounded to the nearest tenth. Use π = 3.14 where necessary.

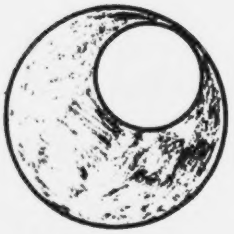
1)



2)



3)



The length of the radius of smaller circle = 5 inches.
The length of the radius of larger circle = 1 foot.

HOMEWORK SOLUTIONS:

1) 55
2) 25π-24 ≈ 54.5
3) 144π - 25π = 119π ≈ 373.7 square inches

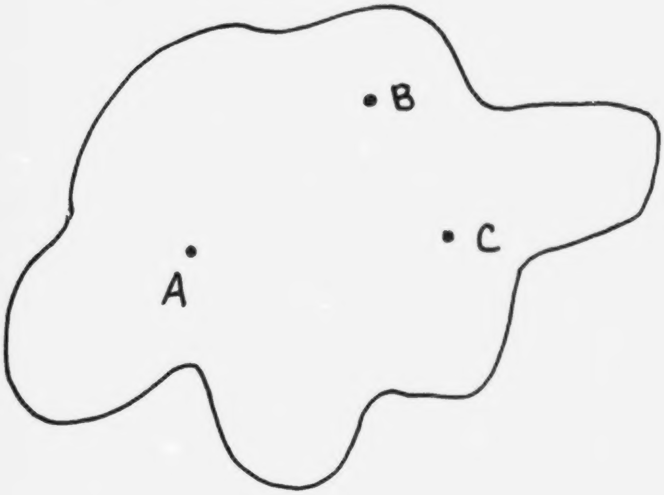
Lesson: 39

AIM: When can three numbers form the lengths of the sides of a triangle?

PERFORMANCE OBJECTIVES: The student will be able to... determine whether a given set of numbers can be the lengths of the three sides of a triangle.

VOCABULARY: the triangle inequality, consecutive odd integers

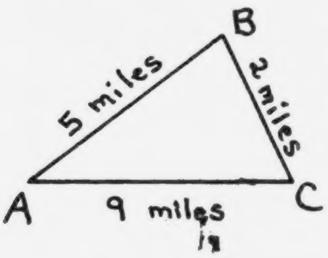
MOTIVATION:



This is a map of a foreign country with three towns: Aqxzv (We will abbreviate it A.), Bbbddp (abbreviation, B) and Csspssc (abbreviation, C). A tourist was told that Aqxzv is 5 miles from Bbbddp, Bbbddp is 2 miles from Csspssc, and Csspssc is 9 miles from Aqxzv.

After studying the situation for a while, the tourist realizes that there must be an error in the information. Can you find the error? Hint: The error is not in the spelling!!

DEVELOPMENT:



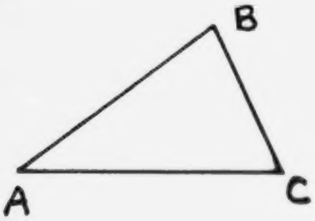
1. Ask the students to draw the "roads" connecting the cities. Say that since this foreign country has no mountains, the roads are straight lines.

2. Elicit that the most direct route between cities A and C should be along $\overline{AC}$. This road is 9 miles long.
   Elicit that the longer route A -> B -> C, according to the diagram, is $5+2 = 7$ miles long, which is impossible!! Therefore, the information had to be wrong. (You might want to cut strips 5 units, 2 units, and 9 units long to further illustrate the point.)
   This means that AB + BC must be more than AC.
   We write $AB + BC > AC$.
3. Say that the problem illustrates a very important result that must be true for all triangles. We call this "the triangle inequality."

   Write the following on the chalkboard.

   > THE TRIANGLE INEQUALITY: In any triangle, the sum of the lengths of any two sides must be more than the length of the third side.

   In symbols:
   $AB + AC > BC$
   $AB + BC > AC$
   $BC + AC > AB$



APPLICATIONS:

Can the following sets of numbers be the lengths of the sides of a triangle?

(1) $\{6, 8, 4\}$

Solution:

$6 + 8 > 4$?	$6 + 4 > 8$	$8 + 4 > 6$?
$14 > 4$?	$10 > 8$?	$12 > 6$?
Yes	Yes	Yes

Answer: Yes, 6, 8 and 4 can be the lengths of the sides of a triangle.

(2) $\{8, 3, 5\}$ (No, since 5 + 3 is NOT > 8.)
(3) $\{11, 5, 5\}$ (No, since 5 + 5 is NOT > 11.)
(4) $\{11, 5, 5.99\}$ (No)
(5) $\{11, 5, 6.001\}$ (Yes)
(6) $\{10, 10, 10\}$ (Yes)

SUMMARY:

1. How can you tell if three numbers can be the lengths of the sides of a triangle?
2. Name three integers that can be the lengths of the sides of an isosceles triangle. Are there any triples of integers, where two are the same, that cannot be the lengths of the sides of an isosceles triangle? Why? Why not?
3. Name three consecutive odd integers that can be the lengths of the sides of a scalene triangle. Are there any such triples that cannot be the lengths of the sides of a scalene triangle? Why? Why not?
4. Name three integers that can be the lengths of the sides of an equilateral triangle. Are there any triples, where all the integers are the same, that cannot be the lengths of the sides of an equilateral triangle? Why? Why not?

HOMEWORK:

In examples 1-5, determine if the set of numbers can be the lengths of the sides of a triangle.

(1) $\{7, 11, 12\}$
(2) $\{8, 3, 4\}$
(3) $\{6.2, 3.7, 5.8\}$
(4) $\{6, 4, 10\}$
(5) $\{3, 10, 7\}$

In examples 6 and 7, what are the possible values of X if the three numbers can be the lengths of the sides of a triangle?

(6) $\{X, 5, 12\}$
(7) $\{3, X, 15\}$

HOMEWORK SOLUTIONS:

(1) Yes

(2) No

(3) Yes

(4) No

(5) No

(6) $7 < X < 17$

(7) $12 < X < 18$

Lesson: 40L

AIM: How do we write a program that will determine if three numbers can form the lengths of the sides of a triangle?

PERFORMANCE OBJECTIVES: The students will be able to ...

1. analyze, plan and code a solution to have the computer determine if three numbers can be the lengths of the sides of a triangle;
2. enter, run and correct his/her program.

MOTIVATION:

Which of the following can represent the lengths of the sides of a triangle?

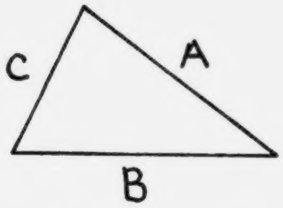
1) {8, 6, 3}
2) {5, 3, 8}
3) {7, 1, 5}
4) {6, 6, 1}

DEVELOPMENT:

1. Say to the students, "Wouldn't it be wonderful if the computer could figure out the answers to the four problems we just went over?"
   We will have the computer solve the following problem.

Problem:	Enter three numbers. Have the computer determine if these numbers can represent the lengths of the sides of a triangle.

2. Elicit the following analysis.
   Analysis



a) In order for A, B, and C to form the lengths of the sides of a triangle, we have three conditions that must hold:

$A+B>C$
and $A+C>B$
and $B+C>A$.

b) Demonstrate that <u>all</u> three inequalities must hold by using examples 2 and 3 from the motivation. (In each case, two of the three inequalities hold but a third one fails.)

c) Point out that often in computer programming, it is easier to test the "opposite" situation. Build up the following chart.

Situation	Opposite Situation
A,B,C WILL form a triangle if	A,B,C will NOT form a triangle if
$A+B>C$ and $A+C>B$ and $B+C>A$	$A+B\le C$ or $A+C\le B$ or $B+C\le A$

Ask, "Why is it easier to test the opposite situation?" Elicit that it is easier to see if ONE situation happens than three at the same time.

3. Elicit the following plan.

<u>Plan</u>

```
1) Enter values for A,B, and C.
2) Check:  Is $A\le 0$?
   a) If yes, print ALL THREE MUST BE POSITIVE.  Go to 1.
   b) If no, continue.
3) Check:  Is $B\le 0$?
   a) If yes, go to 2a.
   b) If no, continue.
4) Check:  Is $C\le 0$?
   a) If yes, go to 2a.
   b) If no, continue.
5) Check:  Is $A+B\le C$?
   a) If yes, print NOT A TRIANGLE.  Go to 1.
   b) If no, continue.
6) Check:  Is $A+C\le B$?
```

   - a) If yes, go to 5a.
   - b) If no, continue.

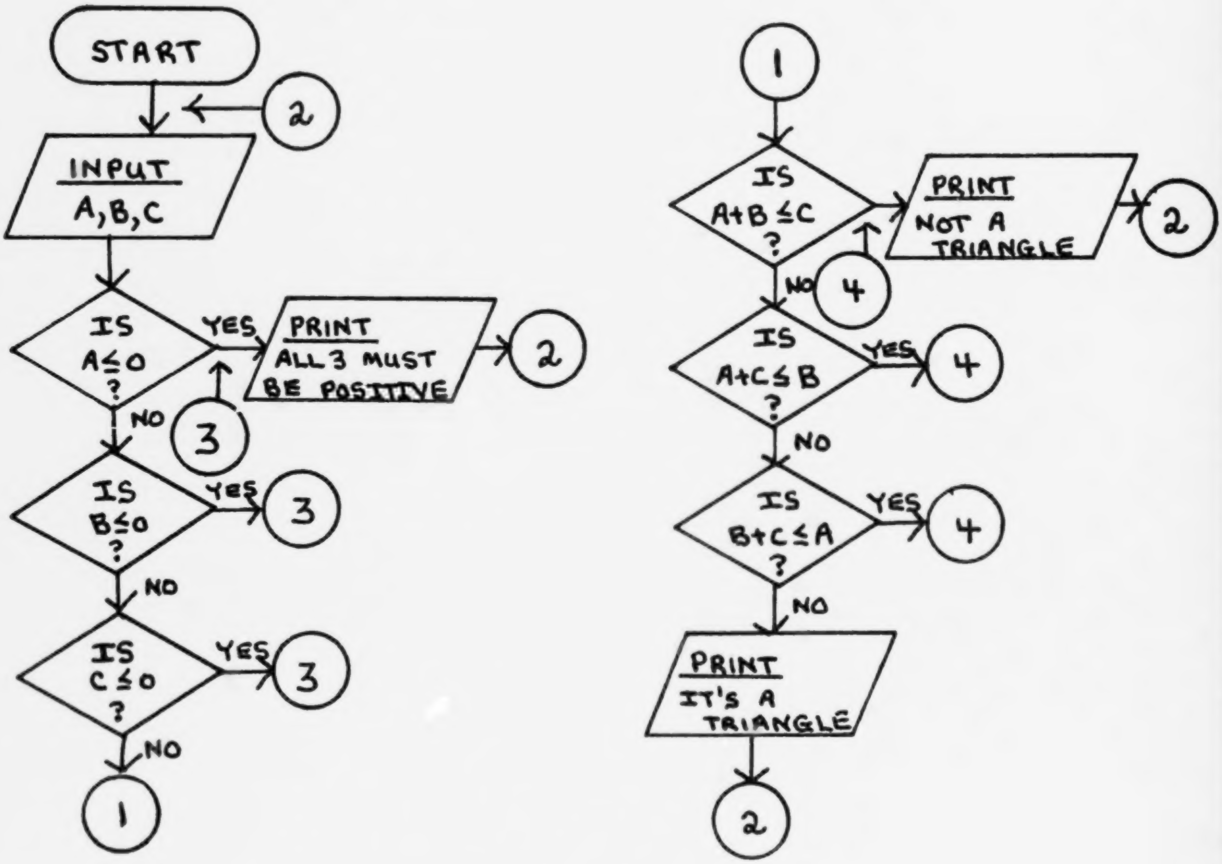
7) Check: Is $B+C\underline{<}A$?
   - a) If yes, go to 5a.
   - b) If no, continue.

8) Print IT'S A TRIANGLE.

9) Go to 1.

4. Ask the students to develop the flowchart which follows.

Flowchart


START
2
INPUT
A,B,C
IS
A≤0
?
YES
PRINT
ALL 3 MUST
BE POSITIVE
2
3
NO
IS
B≤0
?
YES
3
NO
IS
C≤0
?
YES
3
NO
1
1
IS
A+B≤C
?
PRINT
NOT A
TRIANGLE
2
4
NO
IS
A+C≤B
?
YES
4
NO
IS
B+C≤A
?
YES
4
NO
PRINT
IT'S A
TRIANGLE
2

APPLICATION:

Ask the students to design their own planned outputs and to code the program. Circulate to help those having difficulty and to give support to the others. Send students to the computers to enter, RUN and correct the programs.

One possible program follows.

Program

```
10  PRINT "DO THE SIDE LENGTHS FORM A TRIANGLE?"
20  PRINT
30  INPUT "ENTER THE THREE LENGTHS"; A,B,C
40  IF A<=0 THEN GOTO 120
50  IF B<=0 THEN GOTO 120
60  IF C<=0 THEN GOTO 120
70  IF A+B<=C THEN GOTO 160
80  IF A+C<=B THEN GOTO 160
90  IF B+C<=A THEN GOTO 160
100 PRINT "CONGRATULATIONS--IT'S A TRIANGLE!"
110 GOTO 20
120 PRINT "YOU MUST BE MORE CAREFUL."
130 PRINT "YOU CANNOT ENTER A NEGATIVE NUMBER OR ZERO FOR
A LENGTH."
140 PRINT "TRY AGAIN."
150 GOTO 20
160 PRINT "TOO BAD--IT'S NOT A TRIANGLE!"
170 GOTO 20
180 END
```

SUMMARY:

Why did we test for the "opposite situation" in this program?

HOMEWORK:

1) Which of the following lengths form a triangle?
   a) {16, 8, 9} b) {3, 3, 3} c) {5, 10, 15}
   d) {3, 12, 7} e) {8, 12.2, 4.3} f) {10, 8, 1}
2) Revise today's program so that every test uses the situation opposite to the way you did it in class.

HOMEWORK SOLUTIONS:

1a) a triangle
b) a triangle
c) not a triangle
d) not a triangle
e) a triangle
f) not a triangle

2) Program

```
10  PRINT "DO THE SIDE LENGTHS FORM A TRIANGLE?"
20  PRINT
30  INPUT "ENTER THE THREE LENGTHS"; A,B,C
40  IF A>0 THEN GOTO 70
50  PRINT "YOU MUST BE MORE CAREFUL-ENTER POSITIVES
ONLY."
60  GOTO 20
70  IF B>0 THEN GOTO 90
80  GOTO 50
90  IF C>0 THEN GOTO 110
100 GOTO 50
110 IF A+B>C THEN GOTO 140
120 PRINT "TOO BAD-NOT A TRIANGLE."
130 GOTO 20
140 IF A+C>B THEN GOTO 160
150 GOTO 120
160 IF B+C>A THEN GOTO 180
170 GOTO 120
180 PRINT "CONGRATULATIONS-IT'S A TRIANGLE!"
190 GOTO 20
200 END
```

Lesson: 41

AIM: How can we revise the triangle inequality program using the logical operators... AND, OR?

PERFORMANCE OBJECTIVES: The student will be able to ...

1. identify AND and OR as logical operators;
2. trace programs using AND and OR;
3. write programs containing logical operators.

VOCABULARY: logical operator

MOTIVATION:

Which of the following triples can represent the lengths of the sides of a triangle?

1. (15,18,34)   2. (8,6,2)   3. (9,11,14)

DEVELOPMENT:

1. Review with the students that three numbers can form the lengths of the sides of a triangle only if the sum of the lengths of any two sides is greater than the length of the third. Elicit that only the third triple can represent the lengths of the sides of a triangle.
2. Discuss that the program in the previous night's homework question #2 (Lesson 40L) required that we test each of the three possible sums in three separate IF-THEN statements. For example:

```
110 IF A+B > C THEN GOTO 140
140 IF A+C > B THEN GOTO 160
160 IF B+C > A THEN GOTO 180
```

   It is possible to combine "IF-THEN" statements by using LOGICAL OPERATORS. We can write:

```
IF A+B > C AND A+C > B AND B+C > A THEN GOTO 180
```

The operator "AND" causes control to pass to line 180 only when *each* of the conditions is true, that is only when

$A+B > C$ *and*
$A+C > B$ *and*
$B+C > A$.

3. Ask the students to complete Applications problems 1-3.
4. Ask, "How do you think 'OR' would operate in an IF-THEN statement; e.g.,
   `IF D > 10 OR E > 8 THEN GOTO 100?"`
   Elicit that control will pass to line 100 if either condition is true, that is if *either* $D > 10$ *or* $E > 8$.
5. In our triangle example, let us say that line 180 is as follows:
   `180 PRINT "NO SUCH TRIANGLE EXISTS."`
   Ask "What statement using 'OR' would pass control to line 180?" (You may wish to remind the students that if any one sum of lengths is less than or equal to the third length, there is no triangle.) Elicit the response
   `IF A+B < = C OR A+C < = B OR B+C < = A THEN GOTO 180.`
6. Ask the students to complete Applications problems 4-7.

APPLICATIONS:

Questions 1, 2, and 3 refer to this program.

PROGRAM 1

```
10   INPUT X,Y
20   IF X > 70 AND Y > 5 THEN GOTO 50
30   PRINT "NO SWIMMING TODAY."
40   GOTO 60
50   PRING "YOU MAY SWIM TODAY."
60   END
```

1. What message will be displayed if 60 and 4 are input for X and Y?
2. What will be displayed if 60 and 6 are input?
3. What is the output if 80 and 6 are input?

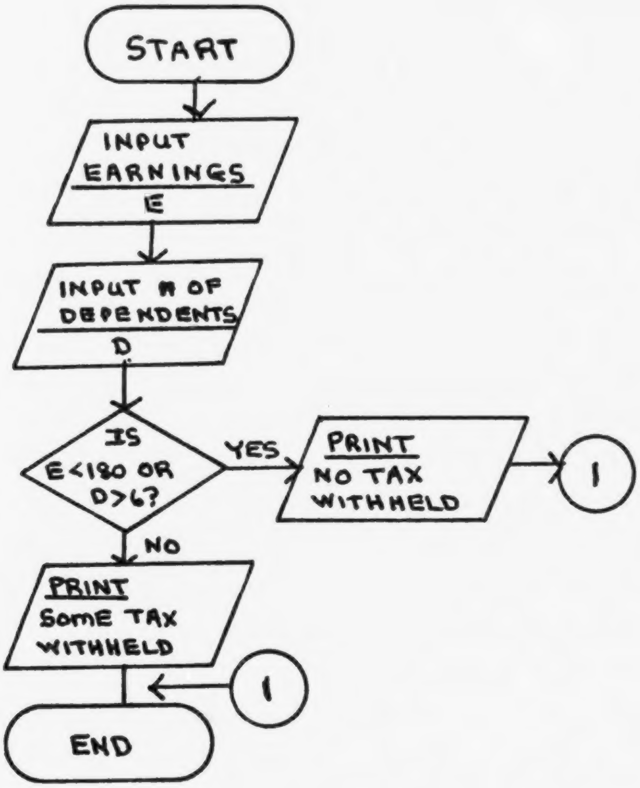
Questions 4,5, and 6 refer to this program.

PROGRAM 2

```
10   INPUT T,I
20   IF T < 20 OR I < 5 THEN GOTO 50
30   PRINT "SKI TRAIL IS OPEN."
40   GOTO 60
50   PRINT "NO SKIING TODAY."
60   END
```

4. What message will be displayed if 40 and 4 are input for T and I?
5. What will be displayed if 30 and 6 are input?
6. What will be displayed if 18 and 8 are input?
7. Employees of the CBA company who earn less than $180 per week or have more than 6 dependents have no income taxes withheld from their salary. Write the program code corresponding to the flowchart below.

Flowchart


START
INPUT EARNINGS E
INPUT # OF DEPENDENTS D
IS E<180 OR D>6?
YES
PRINT NO TAX WITHHELD
1
NO
PRINT SOME TAX WITHHELD
1
END

Summary:

1. Under what circumstances will control pass to line 100 in the following?

```
10  INPUT A,B
20  IF A+B > 10 AND A*B > 10 THEN GOTO 100
```

2. Under what circumstances will control pass to line 100 in the following?

```
10  INPUT X,Y
20  IF X > 10 OR Y > 10 THEN GOTO 100
```

HOMEWORK:

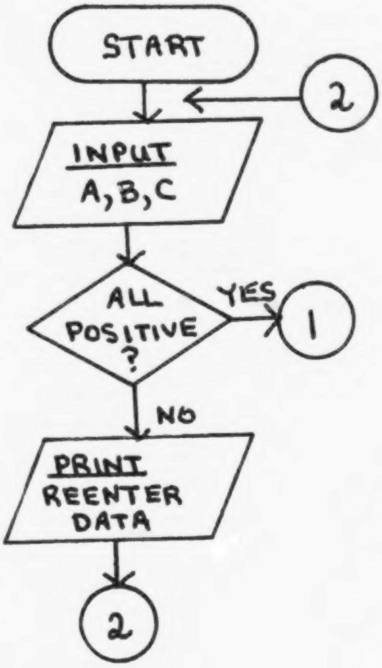
1. Consider PROGRAM 1 in Applications problems 1-3. What message will be displayed if the following are input for X and Y?
   a. 100 and 6
   b. 50 and 4
   c. 75 and 3
   d. 65 and 10
   e. 70 and 5
2. Consider PROGRAM 2 in Applications Problems 4-6. What will be displayed if the following are input for T and I.
   a. 15 and 6
   b. 21 and 6
   c. 30 and 4
3. Revise the program in Applications problem 7 so that some tax is withheld if E > 200 and D < 7. Use the AND operator.
4. Many of our programs require that we check if all the input values are positive. Write a flowchart and program code for that part of a program in which three variables are input; if any are not positive, the message "REENTER DATA." is displayed and more values are input. If all the values are positive, control passes to line 100. Use the logical operator AND.
5. Repeat problem 4, except use the operator OR.

HOMEWORK SOLUTIONS:

1. a. YOU MAY SWIM TODAY.
   b. NO SWIMMING TODAY.
   c. NO SWIMMING TODAY.
   d. NO SWIMMING TODAY.
   e. NO SWIMMING TODAY.
2. a. NO SKIING TODAY.
   b. SKI TRIAL IS OPEN.
   c. NO SKIING TODAY.
3.

```
10   INPUT E
20   INPUT D
30   IF E > 200 AND D < 7 THEN GOTO 60
40   PRINT "NO TAX WITHHELD."
50   GOTO 70
60   PRINT "SOME TAX IS WITHHELD."
70   END
```

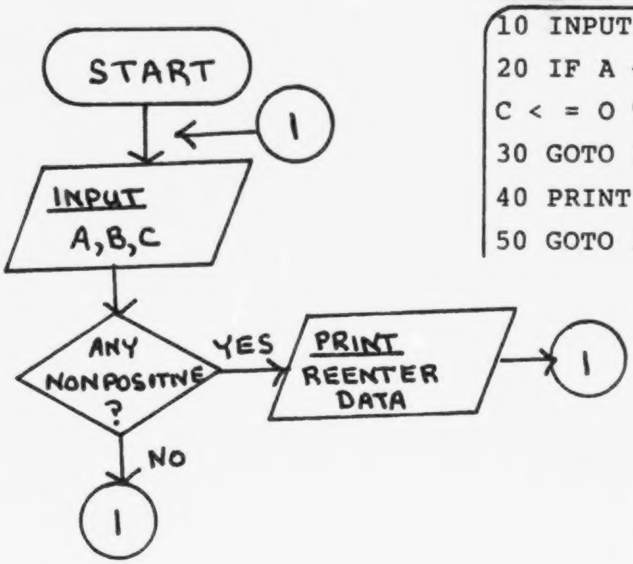
4. Flowchart


START
2
INPUT
A,B,C
ALL
POSITIVE
?
YES
1
NO
PRINT
REENTER
DATA
2

Program

```
10 INPUT A,B,C
20 IF A > 0 AND B > 0 AND
C > 0 THEN GOTO 100
30 PRINT "REENTER DATA."
40 GOTO 10
```

5. Flowchart


START
1
INPUT
A,B,C
ANY
NONPOSITIVE
?
YES
PRINT
REENTER
DATA
1
NO
1

Program

```
10 INPUT A, B, C
20 IF A < = 0 OR B < = 0 OR
C < = 0 THEN GOTO 40
30 GOTO 100
40 PRINT "REENTER DATA."
50 GOTO 10
```

Lesson: 42

AIM: What is the relationship among the sides and the angles of a triangle?

PERFORMANCE OBJECTIVES: The student will be able to...

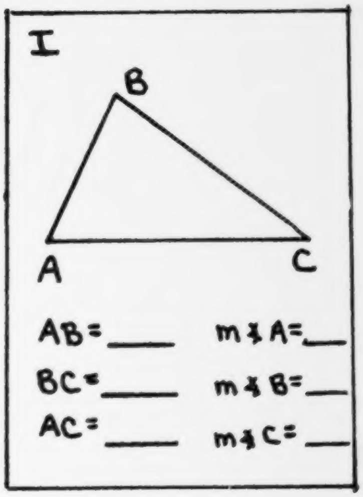
1. recognize that in any triangle, the longest (shortest) side is opposite the largest (smallest) angle and vice versa;
2. recognize that if two sides of a triangle are congruent, the angles opposite these sides are congruent and vice versa;
3. recognize that an equilateral triangle is equiangular.

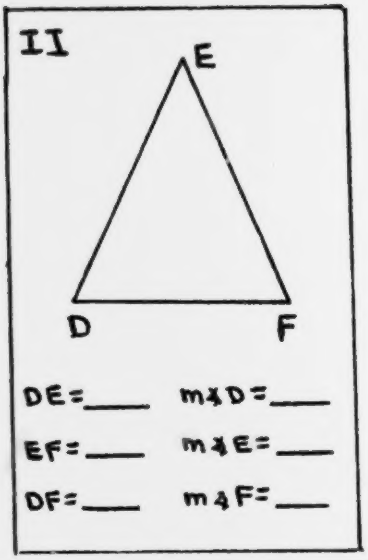
VOCABULARY: hypotenuse, legs, equilateral triangle, equiangular triangle, scalene triangle, isosceles triangle, base, base angles, vertex angle, obtuse angle, acute angle, right angle, congruent, right triangle, noncongruent, obtuse triangle, acute triangle.

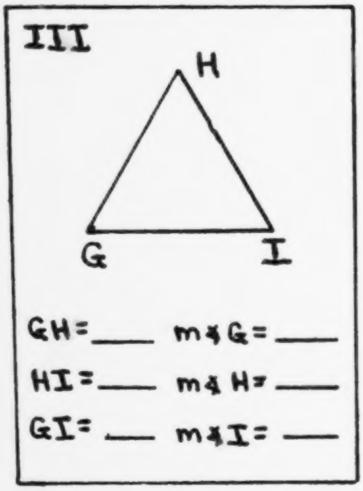
Symbols: $\triangle ABC$ means triangle ABC.
$\overline{AB}$ means line segment AB.
AB means the length of line segment AB.
$m\angle A$ means the measure of angle A.

MOTIVATION:

1. Divide the students in your class into groups of four.
2. Distribute to each group three prepared rexograph stencils picturing enlarged versions of the scalene, isosceles, and equilateral triangles which follow.







3. The students will also need protractors and rulers (preferably metric).

4. Ask the students to find and record the lengths of the sides and the measures of the angles for each triangle. Circulate around the room and assist with the use of the rulers and protractors where needed.

DEVELOPMENT:

1. Directing the students' attention to rexograph I, ask them to notice that the sides and angles all have different measures. Ask, "What do we call a triangle whose sides have different lengths? Which is the longest side of triangle ABC? How long is it? Which angle of triangle ABC has the largest measure? What is its measure? Relative to the largest angle of triangle ABC, where do we find the longest side? Which is the shortest side of triangle ABC? How long is it? Which angle of triangle ABC has the smallest measure? What is its measure? Relative to the smallest angle of triangle ABC, where do we find the shortest side? Generally, how are the sides and angles of a triangle related?"

2. Directing the students' attention to rexograph II, ask them to notice that two of the sides and two of the angles have the same measures. Ask, "What do we call a triangle which has two sides that are equal in length? Which sides of triangle DEF are equal in length or congruent? What is their common length? What name is given to the two congruent sides of an isosceles triangle? ... the noncongruent side? Which angles of triangle DEF are equal in measure or congruent? What is their common measure? What name is given to the congruent angles in an isosceles triangle? ... the noncongruent angle? Relative to the congruent angles, where do we find the congruent sides?"

3. Directing the students' attention to rexograph III, ask them to notice that all three sides have the same length. Ask, "What name is given to a triangle all of whose sides have the same length? What is true about all the angles of the equilateral triangle GHI?" Say to the students that we say that equilateral triangle GHI is also equiangular.

4. Ask the students to complete the Applications problems. Ask them to draw the appropriate diagrams on the chalkboard for each.

APPLICATIONS:

1. If triangle ABC is an obtuse triangle with angle B the obtuse angle, which is the longest side of the triangle?

2a. Which angle is always the largest in a right triangle?

2b. Which side is the longest in a right triangle? Say to the students that the side opposite the right angle in a right triangle is called the hypotenuse. Ask the students to note that the hypotenuse is the longest side in any right triangle.

3. In isosceles triangle ABC, $\overline{AB}$ is congruent to $\overline{AC}$. Which angles are congruent?

In 4-10, if the blank space is replaced by the word **always**, **sometimes**, or **never**, the resulting statement will be true. Select the word which will correctly complete the statement.

4. In an isosceles obtuse triangle, the base angles are ...... acute.
5. The number of degrees in each angle of an equilateral triangle is ...... the same.
6. The perimeters of equilateral triangles are ...... the same.
7. The measures of the angles of a scalene triangle are ...... the same.
8. In triangle ABC, if the length of $\overline{AB}$ is greater than the length of $\overline{AC}$, then the measure of angle C is ...... greater than the measure of angle B.
9. In triangle ABC, the measure of angle A is more than sixty. Side $\overline{BC}$ is ...... the longest side of the triangle.
10. If one of the congruent sides of an isosceles triangle is longer than the base, then the measure of the angle opposite the base is ...... greater than sixty.

SUMMARY:

1. What is the relationship among the sides and the angles in a scalene triangle?
2. What is the relationship among the sides and the angles in an isosceles triangle?
3. What is the relationship among the sides and the angles in an equilateral triangle?

HOMEWORK:

1. In right triangle XYZ, angle Y is the right angle. Name the longest side of the triangle.

2. Tell which length would be the measurement of the hypotenuse of a right triangle, the lengths of whose sides are ...

   a) 3', 4', 5'
   b) 5", 13", 12"
   c) 8 cm, 10 cm, 6 cm
   d) 26 mm, 24 mm, 10 mm

3. In triangle XYZ, $XY=5$ cm, $YZ=6$ cm, and $XZ=4$ cm. Name the angle of the triangle which has the greatest measure.

4. In triangle ABC, $m<A=45$, $m<B=60$ and $m<C=75$. Name the side of the triangle which has the greatest measure.

5. In triangle RST, $m<R=55$ and $m<S=70$.
   a) Find the measure of angle T.
   b) Name the legs of triangle RST.
   c) Classify the triangle according to the measures of both its sides and angles.

6. In triangle ABC, $m<C=90$ and $m<B=35$.
   a) Name the shortest side of this triangle.
   b) Classify the triangle according to the measures of both its sides and angles.

7. In triangle WXY, $m<W=125$ and $m<X=25$.
   a) What is the measure of angle Y?
   b) Which is the longest side of triangle WXY?... the shortest side of triangle WXY?
   c) Classify the triangle according to the measures of both its sides and angles.

HOMEWORK SOLUTIONS:

1. $\overline{XZ}$

2a) 5'   c) 10 cm
b) 13"   d) 26 mm

3. <X

4. $\overline{AB}$

5a) 55

b) $\overline{SR}$, $\overline{ST}$
c) Isosceles acute triangle

6a) $\overline{AC}$
b) Scalene right triangle

7a) 30
b) The longest side is $\overline{XY}$; the shortest side is $\overline{WY}$.
c) Scalene obtuse triangle

Lesson: 43L

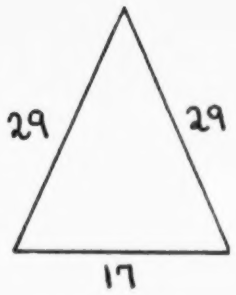
AIM: How do we code a program to determine if a triangle is isosceles, equilateral or scalene, given the measures of three angles?

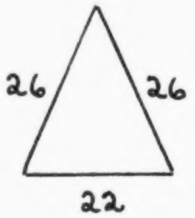
PERFORMANCE OBJECTIVES: The student will be able to ...

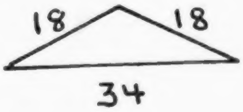
1. develop an algorithm to determine if a triangle is isosceles, equilateral or scalene, given the measures of three angles;
2. incorporate his/her algorithm into a flowchart;
3. design an output for the program;
4. write a program in BASIC to determine if a triangle is isosceles, equilateral or scalene.

VOCABULARY: isosceles, equilateral, scalene, legs, base angles, vertex angle, congruent

MOTIVATION:







If the numbers above represent the measures of the sides of the triangles, what do the triangles have in common?

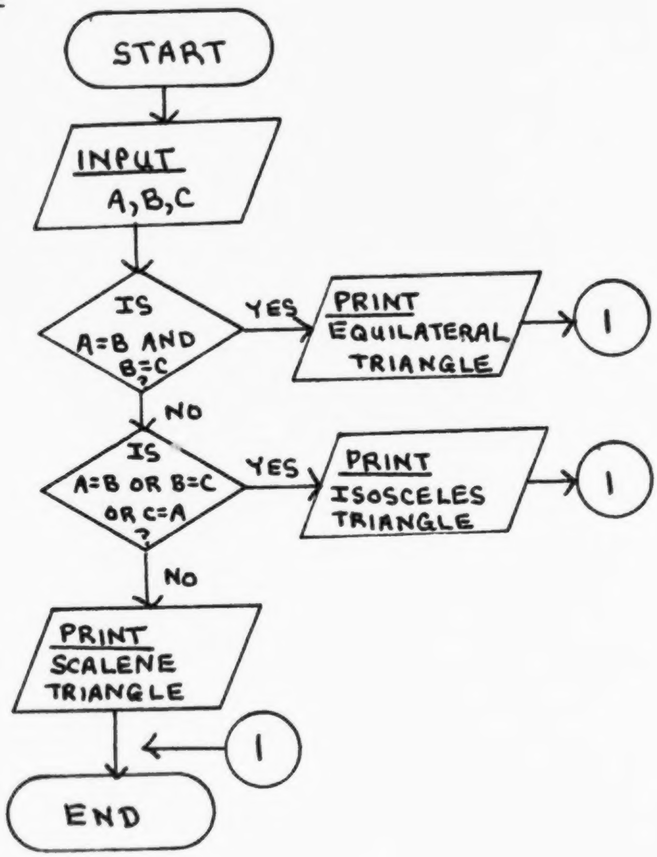
DEVELOPMENT:

1. Elicit from the students that each triangle is isosceles since each has two sides of equal length. Ask the students what must be true concerning the angles of an isosceles triangle. (The base angles must have equal measures.) Elicit the following chart.

TRIANGLE	NUMBER OF CONGRUENT ANGLES
EQUILATERAL	3
ISOSCELES	2
SCALENE	0

2. Ask the students, "Given three angle measures, how can we determine if the triangle is equilateral, isosceles, or scalene?" Elicit that we can test to see if the three angles are congruent (equal in measure), and therefore, we would have an equilateral triangle. If not, we can test to see if any two angles are congruent, leading to an isosceles triangle. Ask, "If not, what type of triangle must we have?" Elicit a scalene triangle.
3. Ask the students to prepare a flowchart based on the algorithm above.

Flowchart


START
INPUT
A,B,C
IS A=B AND B=C ?
YES
PRINT EQUILATERAL TRIANGLE
1
NO
IS A=B OR B=C OR C=A ?
YES
PRINT ISOSCELES TRIANGLE
1
NO
PRINT SCALENE TRIANGLE
1
END

4. Ask "Can the first decision in the flowchart be simplified?"
   Elicit that if A=B <u>and</u> B=C, then C=A automatically and we need not include C=A in the IF-THEN statement.

APPLICATIONS:

1. Ask the students to write the program code based on the flowchart and their own planned output.

<u>Possible Planned Output</u>

```
CLASSIFYING TRIANGLES
INPUT 3 ANGLE MEASURES.
? 60,40,80
THE TRIANGLE IS SCALENE.
RUN
CLASSIFYING TRIANGLES
INPUT 3 ANGLE MEASURES.
? 60,60,60
THE TRIANGLE IS EQUILATERAL.
RUN
CLASSIFYING TRIANGLES
INPUT 3 ANGLE MEASURES.
? 45,90,45
THE TRIANGLE IS ISOSCELES.
```

<u>Program</u>

```
10     PRINT "CLASSIFYING TRIANGLES"
20     PRINT "INPUT 3 ANGLE MEASURES."
30     INPUT A,B,C
40     IF A=B AND B=C THEN GOTO 80
50     IF A=B OR B=C OR C=A THEN GOTO 100
60     PRINT "THE TRIANGLE IS SCALENE."
70     GOTO 110
80     PRINT "THE TRIANGLE IS EQUILATERAL."
90     GOTO 110
100    PRINT "THE TRIANGLE IS ISOSCELES."
110    END
```

2. Allow the students who have written the program to enter and run it on the computers.
3. Circulate around the laboratory assisting those students needing help. Grouping students by "need" may be useful. Students who have completed their work on the computers can assist others.
4. Allow the students (working in pairs if students outnumber computer terminals) to enter the programs written in previous homework assignments on the computers. Ask them to verify that the programs are correct.

SUMMARY:

Summarize the lesson by asking the students to write the program to classify triangles according to the measures of their sides on the chalkboard.

HOMEWORK:

1. Revise the program written in class so that it will produce the following output.

Planned Output

```
WHAT KIND OF TRIANGLE IS IT?
ENTER THE MEASURE OF EACH OF THE 3 ANGLES.
? 35,45,100
IT IS A SCALENE TRIANGLE.
WHAT KIND OF TRIANGLE IS IT?
ENTER THE MEASURE OF EACH OF THE 3 ANGLES.
? 60,60,60
IT IS AN EQUILATERAL TRIANGLE.
WHAT KIND OF TRIANGLE IS IT?
ENTER THE MEASURE OF EACH OF THE 3 ANGLES.
? 15,150,15
IT IS AN ISOSCELES TRIANGLE.
WHAT KIND OF TRIANGLE IS IT?
ENTER THE MEASURE OF EACH OF THE 3 ANGLES.
?
```

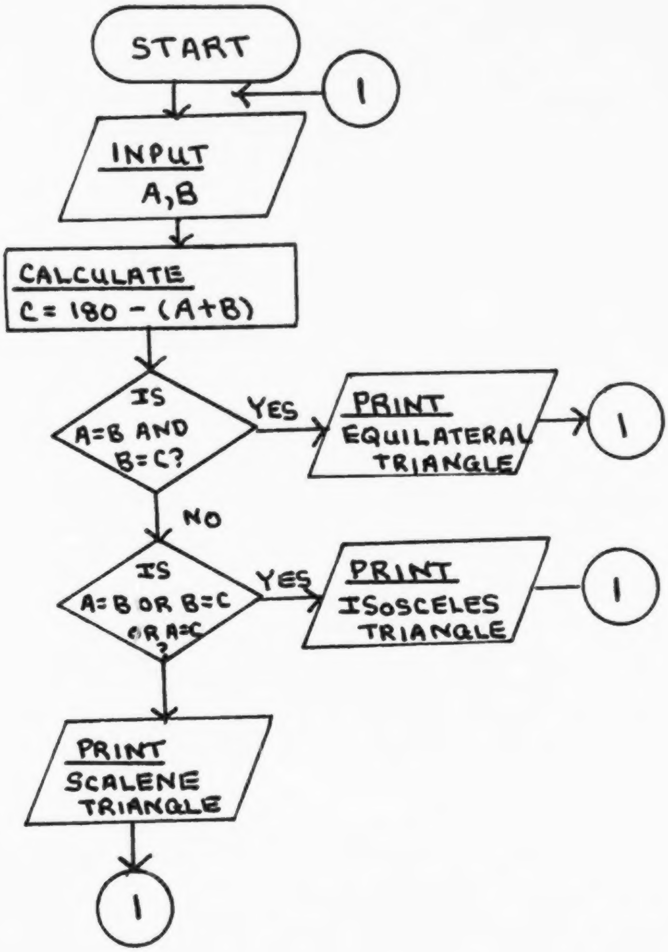
2. Develop a flowchart, design an output and write a program to have the computer determine the kind of triangle (scalene, isosceles, equilateral) if the measures of only <u>two</u> angles of the triangle are input.

HOMEWORK SOLUTIONS:

1. Change in ...

```
line 10:   10  PRINT "WHAT KIND OF TRIANGLE IS IT?"
line 20:   20  PRINT "ENTER THE MEASURE OF EACH OF THE 3
           ANGLES."
line 60:   60  PRINT "IT IS A SCALENE TRIANGLE."
line 70:   70  GOTO 10
line 80:   80  PRINT "IT IS AN EQUILATERAL TRIANGLE."
line 90:   90  GOTO 10
line 100:  100 PRINT "IT IS AN ISOSCELES TRIANGLE.":GOTO 10
```

2. <u>Flowchart</u>


START
1
INPUT
A,B
CALCULATE
C = 180 - (A+B)
IS
A=B AND
B=C?
YES
PRINT
EQUILATERAL
TRIANGLE
1
NO
IS
A=B OR B=C
OR A=C
?
YES
PRINT
ISOSCELES
TRIANGLE
1
PRINT
SCALENE
TRIANGLE
1

Planned Output

```
IS THE TRIANGLE ISOSCELES,
SCALENE OR EQUILATERAL?
TWO ANGLE MEASURES PLEASE? 60,40
SCALENE
TWO ANGLE MEASURES PLEASE? 60,60
EQUILATERAL
TWO ANGLE MEASURES PLEASE? 15,150
ISOSCELES
TWO ANGLE MEASURES PLEASE?
```

Program

```
10  PRINT "IS THE TRIANGLE ISOSCELES,"
20  PRINT "SCALENE OR EQUILATERAL?"
30  INPUT "TWO ANGLE MEASURES PLEASE"; A,B
40  C=180-(A+B)
50  IF A=B AND B=C THEN 90
60  IF A=B OR B=C OR C=A THEN 110
70  PRINT "SCALENE"
80  GOTO 30
90  PRINT "EQUILATERAL"
100 GOTO 30
110 PRINT "ISOSCELES"
120 GOTO 30
130 END
```

Lesson: 44

AIM: What is the Pythagorean relationship?

PERFORMANCE OBJECTIVES: The student will be able to ...

1. identify the hypotenuse of a right triangle;
2. state the Pythagorean relationship;
3. determine the length of one side of a right triangle, given the lengths of the other two sides.

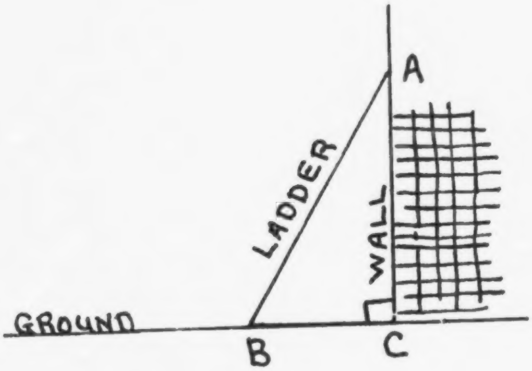
VOCABULARY: hypotenuse, legs, Pythagorean relationship, perpendicular

MOTIVATION:

If the base of a ladder is placed 5 feet away from a wall, how long must it be to reach a point 12 feet above ground level?

DEVELOPMENT:

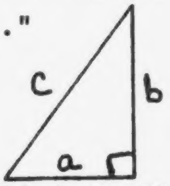
1. Ask, "What geometric figure is formed by the ladder, the wall and the ground?" Elicit that a right triangle is formed (if the wall is perpendicular to the ground). Ask a student to draw a diagram on the chalkboard; e.g.,



2. Ask, "Which is the longest side of triangle ABC? How do we know it is the longest? What name is given to this side? What name is given to the two shorter sides of triangle ABC?"

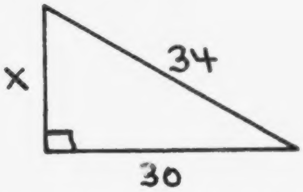
3. Say to the students that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. This is known as the Pythagorean relationship.

4. Say to the students, "Let us redraw the diagram, using a,b, and c as the lengths of the sides."



$c^2=a^2+b^2$

5. Ask, "How can this relationship be applied to find the length of the ladder?" Elicit that $c^2=5^2+12^2$ or $c^2=169$, $c=\sqrt{169}$ and c=13 feet.
6. Ask the students to complete Applications example 1.
7. Ask the students to consider the triangle below and find the value of x.



Point out that the Pythagorean relationship can be applied if the length of a leg is unknown.

Elicit that

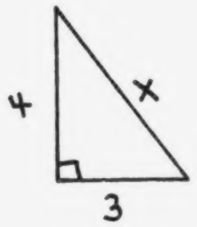
$$\begin{aligned} 34^2&=x^2+30^2\\ 1156&=x^2+\cancel{900}\\ \underline{-900}&\quad \underline{-\cancel{900}}\\ 256&=x^2\\ \sqrt{256}&=x \text{ or } x=16. \end{aligned}$$

8. Ask the students to complete Applications problem 4.
9. Ask the students to complete all the other Applications problems not yet completed.

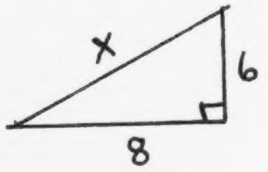
APPLICATIONS:

In problems 1 through 6, solve for x.

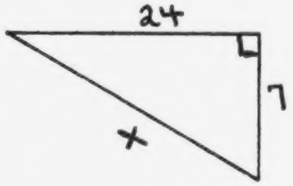
1.



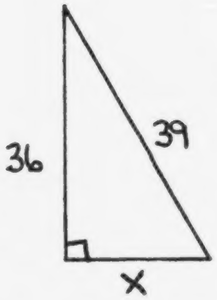
2.



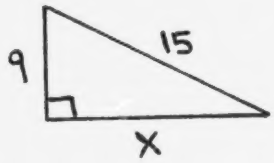
3.



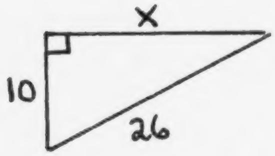
4.



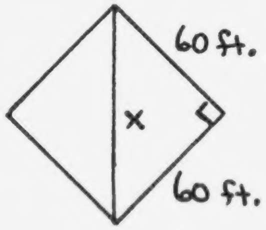
5.



6.



7. On a little league baseball field, the bases are 60 feet apart. Find, to the nearest whole number, the distance from home plate to second base.



8. If c= a^2+b^2, write this statement in BASIC.

SUMMARY:

1. If the lengths of the legs of a right triangle are represented by a and b, and the length of the hypotenuse by c, what relationship exists among a, b and c?
2. If you are given three numbers which represent the lengths of the sides of a right triangle, which number represents the length of the hypotenuse?

HOMEWORK:

In examples 1 through 4, a and b represent the lengths of the legs of a right triangle and c the length of the hypotenuse. Find the value of x.

1. a=8, b=15, c=x
2. a=24, b=18, c=x
3. a=4, b=x, c=5
4. a=x, b=48, c=50

5. What is the length of the longest cane which can be placed on the bottom of a suitcase 1.5' long and 2' wide?

HOMEWORK SOLUTIONS:

1. x=17
2. x=30
3. x=3
4. x=14
5. The length of the longest cane is 2.5 feet.

Lesson: 45

AIM: How do we solve problems involving the use of the Pythagorean relationship?

(The students may use calculators for this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to ...

1. write an equation using the Pythagorean relationship, based on a given problem;
2. solve the equation for the length of the missing side of the triangle.

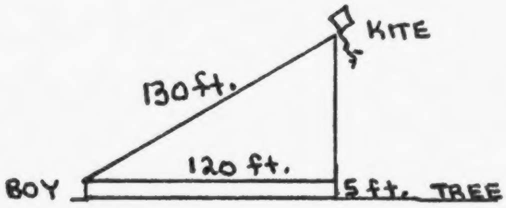
VOCABULARY: Pythagorean relationship, legs, hypotenuse, diagonal, parallel

MOTIVATION:

A kite, flying at the end of a 130 foot long string, is observed directly over a tree 120 feet from the child holding the string. How high is the kite above the ground? (Assume that the kite string is being held 5 feet above the ground.)

DEVELOPMENT:

1. Elicit the diagram below, corresponding to the given problem.



2. Ask, "What relationship exists between x and the given numbers?" ($130^2=x^2+120^2$) Ask the students to solve the equation ($x=50$), determining that the kite is 55 ($x+5$ in the diagram) feet above the ground; e.g.,

$$c^2=a^2+b^2$$
$$130^2=x^2+120^2$$
$$16900=x^2+144\cancel{00}$$
$$-14400 \quad -\cancel{14}400$$
$$2500 = x^2$$
$$2500 = x \text{ or } x=50$$
$$\text{and } x+5=50+5=55 \text{ feet.}$$

3. Remind the students that in problems concerning "length" or "distance", a diagram is very helpful, and that if the diagram includes a right triangle, Pythagoras' relationship may be a useful tool in solving the problem.

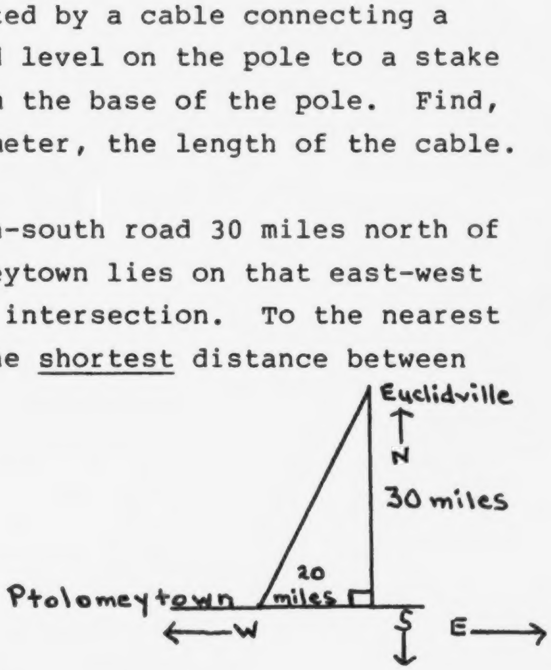
4. Ask the students to find the length of the diagonal of a square whose side measures 4". Elicit that $x^2=4^2+4^2$ since the diagonal of the square is also the hypotenuse of a right triangle whose legs are the sides of the square. Ask the students to solve for x. ($x^2=16+16=32$, $x= 32$) Show that since $5^2=25$ and $6^2=36$, x must be between 5 and 6. Using a table of square roots or a calculator, ask the students to find 32 to the nearest tenth. (5.7)

5. Ask the students to complete the Applications problems.

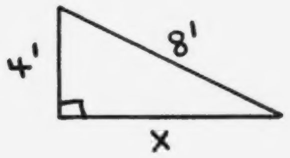
APPLICATIONS:

1. A telephone pole is supported by a cable connecting a point 6 meters above ground level on the pole to a stake in the ground 5 meters from the base of the pole. Find, to the nearest tenth of a meter, the length of the cable.

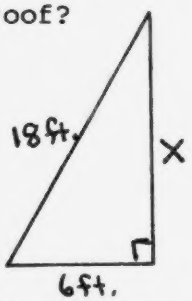
2. Euclidville lies on a north-south road 30 miles north of an east-west road. Ptolomeytown lies on that east-west road, 20 miles west of the intersection. To the nearest tenth of a mile, what is the shortest distance between the 2 towns?



3. An 8 foot long ramp connects the back of a truck to the street. If the back of the truck is 4 feet above the street, how far, to the nearest tenth of a foot, from the truck will the bottom of the ramp reach the ground?



4. An 18 foot ladder reaches the roof of a house when the foot of the ladder is placed 6 feet from the house. How high, to the nearest foot, is the roof?



SUMMARY:

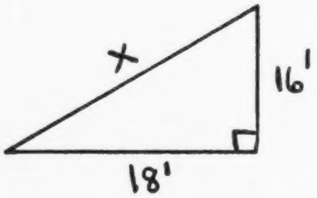
How do we use the Pythagorean relationship to solve problems?

HOMEWORK:

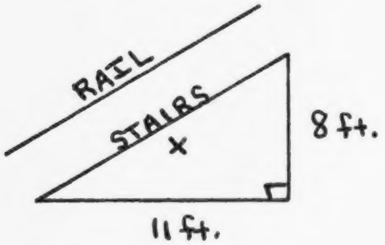
Solve each problem. Use the Newton-Raphson method to approximate the square roots where necessary.

1. After running with his kite, Matt ties the kite string to a branch on the ground. If the kite string is 34 feet long and the kite is directly above a point 16 feet from Matt, how high is the kite off the ground?
2. A gate is made up of several planks of wood resulting in a rectangular gate 5 feet long and 4 feet high. The gate is braced by another plank nailed across a diagonal of the rectangle. To the nearest tenth of a foot, how long is this diagonal brace?

3. A flagpole was struck by lightening at a point 16 feet above the ground. The pole broke but the upper section remained attached to the lower section. However, the top of the pole now rests at a point 18 feet from the base of the pole. Find, to the nearest tenth of a foot, the height of the pole before it broke.



4. A staircase occupies a space 11 feet long on the first floor of the house and rises 8 feet to the second floor. How long, to the nearest tenth of a foot, a handrail must be used if it is to be placed parallel to the flight of stairs?



Note that since the rail is parallel to the flight of stairs, it is as long as the distance along the stairs from the first to the second floor.

HOMEWORK SOLUTIONS:

1. The kite is 30 feet off the ground.
2. The diagonal brace is about 6.4 feet long.
3. The pole was about 40.1 feet long before it broke.
   (24.1+16 = 40.1 ft.)
4. The handrail must be about 13.6 feet long.

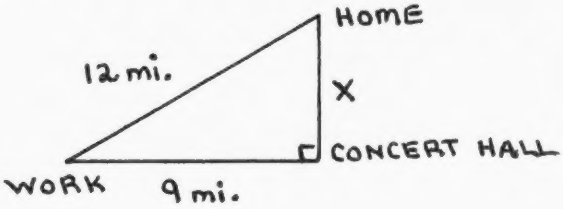
Lesson: 46L

AIM: How do we code a program to find the length of a leg of a right triangle, given the lengths of the hypotenuse and the other leg?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop an algorithm for computing the length of a leg of a right triangle when given the lengths of the hypotenuse and the other leg;
2. prepare a flowchart using the algorithm;
3. design an appropriate output;
4. write a program to compute the length of the leg;
5. enter, run and edit the program.

MOTIVATION: How far is it from the concert hall to home? Round your answer to the nearest tenth of a mile.



Elicit $12^2=x^2+9^2$, $144=x^2+81$, $x^2=63$, $x=\sqrt{63}$ 7.9.

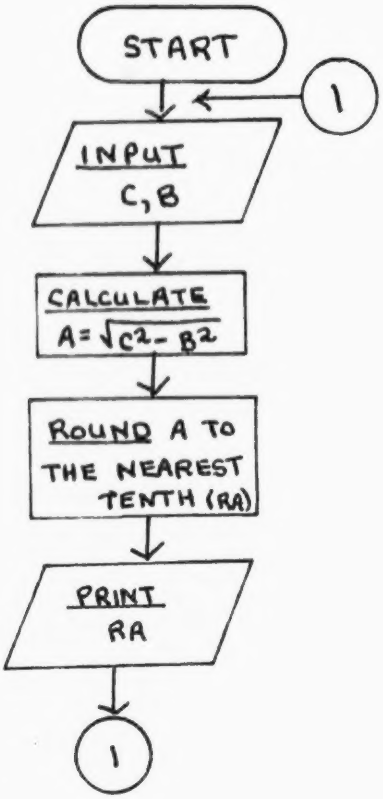
DEVELOPMENT:

1. Ask: "How can we use the computer to solve this problem?" The students should respond that the Pythagorean relationship can furnish the algorithm. If $C^2=A^2+B^2$, then $A^2=C^2-B^2$ and $A=\sqrt{C^2-B^2}$.
2. Elicit a brief outline of the program and ask the students to flowchart the plan.

Plan

1) Input the length of the hypotenuse and given leg: C,B.
2) Compute the length of the missing leg: $A = C^2-B^2$.
3) Round A to the nearest tenth, RA.
4) Display the result, RA.
5) Go to 1.

Flowchart


START
1
INPUT
C,B
CALCULATE
A = √C² - B²
ROUND A TO THE NEAREST TENTH (RA)
PRINT
RA
1

3. Allow the students to design their own outputs, including a heading. One possibility follows.

Planned Output

```
CALCULATING THE LENGTH OF A LEG
(Skip 1 line.)
ENTER THE LENGTHS OF THE HYPOTENUSE
AND GIVEN LEG? 12,9
THE LENGTH OF THE OTHER LEG IS
ABOUT 7.9
(Skip 1 line.)
ENTER THE LENGTHS OF THE HYPOTENUSE
AND GIVEN LEG?
```

APPLICATIONS:

1. Ask the students to write the BASIC program code based on the flowchart and planned output. As the students complete their programs, they can enter and RUN the programs on the computer. Circulate around the room checking on student progress and assisting those encountering difficulty. A possible program follows.

   Program

```
10  PRINT "CALCULATING THE LENGTH OF A LEG"
20  PRINT
30  PRINT "ENTER THE LENGTHS OF THE HYPOTENUSE"
40  INPUT "AND GIVEN LEG"; C,B
50  A=SQR(C↑2-B↑2)
60  N = A+.05:RA = INT(10*N)/10
70  PRINT "THE LENGTH OF THE OTHER LEG IS"
80  PRINT "ABOUT"; RA
90  GOTO 20
100 END
```

2. If time permits, ask the students to modify their programs by changing the output (heading and prompts).

SUMMARY:

1. What are the key ideas in a program to compute the length of the missing leg in a right triangle?

2. When given two numbers representing the lengths of the hypotenuse and the leg of a right triangle, which number will always be input first? Can the numbers be the same?

HOMEWORK:

1. Modify the program written in today's lesson so that it will produce the output which follows.

   Planned Output

```
 FINDING THE MISSING LEG
 (Skip 2 lines.)
 WHAT ARE THE HYPOTENUSE AND KNOWN LEG? 12,9
*THE MISSING LEG MEASURES ABOUT 7.94
 (Skip 2 lines.)
 WHAT ARE THE HYPOTENUSE AND KNOWN LEG?
```

   * Note that the length has been rounded to the nearest hundredth.

2. Develop a flowchart, design an output, and write a program in BASIC to input the lengths of two legs of a right triangle and have the computer compute and display the length of the hypotenuse, rounded to the nearest tenth.

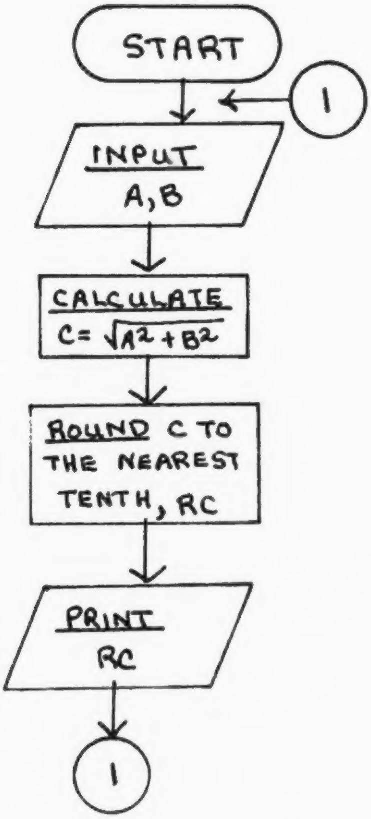
3. Revise the flowchart and planned output and, add a segment to the program in question 2 above that will print "ENTER ONLY POSITIVE NUMBERS" and go to the INPUT statement if the input lengths are zero or negative. Also, round the length of the hypotenuse to the nearest hundredth.

HOMEWORK SOLUTIONS:

1.

```
10 PRINT "FIND THE MISSING LEG"
20 PRINT:PRINT
30 INPUT "WHAT ARE THE HYPOTENUSE AND KNOWN LEG";C,B
40 A=SQR(C↑2-B↑2)
50 N=A+.005:RA=INT(100*N)/100
60 PRINT "THE MISSING LEG MEASURES ABOUT";RA
70 GOTO 20
80 END
```

2. Flowchart


START
1
INPUT
A,B
CALCULATE
$C=\sqrt{A^2+B^2}$
ROUND C TO THE NEAREST TENTH, RC
PRINT
RC
1

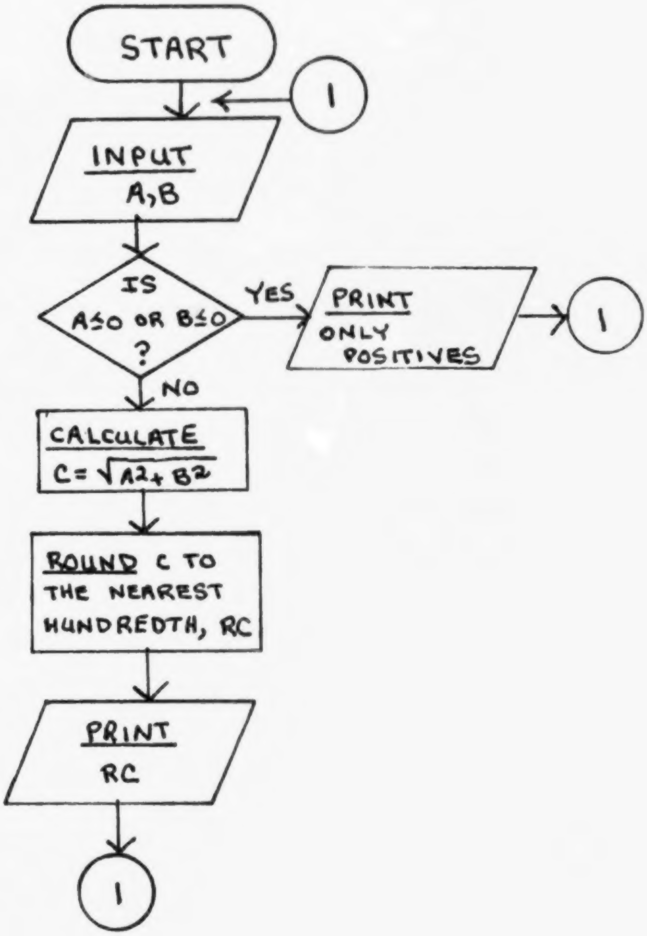
Planned Output

```
FINDING THE LENGTH OF THE HYPOTENUSE
(Skip 2 lines.)
ENTER THE LENGTHS OF THE LEGS.
? 6,5
THE LENGTH OF THE HYPOTENUSE
IS ABOUT 7.8
(Skip 2 lines.)
ENTER THE LENGTHS OF THE LEGS.
?
```

Program

```
10  PRINT "FINDING THE LENGTH OF THE HYPOTENUSE"
20  PRINT:PRINT
30  PRINT "ENTER THE LENGTHS OF THE LEGS."
40  INPUT A,B
50  C=SQR(A↑2+B↑2)
60  N=C+.05:RC=INT(10*N)/10
70  PRINT "THE LENGTH OF THE HYPOTENUSE"
80  PRINT "IS ABOUT";RC
90  GOTO 20
100 END
```

3. Flowchart


START
1
INPUT A,B
IS A≤0 OR B≤0 ?
YES
PRINT ONLY POSITIVES
1
NO
CALCULATE C=$\sqrt{A^2+B^2}$
ROUND C TO THE NEAREST HUNDREDTH, RC
PRINT RC
1

Planned Output

```
FINDING THE LENGTH OF THE HYPOTENUSE
(Skip 2 lines.)
ENTER THE LENGTHS OF THE LEGS.
? 6,5
THE LENGTH OF THE HYPOTENUSE
IS ABOUT 7.81
(Skip 2 lines.)
ENTER THE LENGTHS OF THE LEGS.
? 6,0
ENTER ONLY POSITIVE NUMBERS.
(Skip 2 lines.)
ENTER THE LENGTHS OF THE LEGS.
?
```

Program

```
10  PRINT "FINDING THE LENGTH OF THE HYPOTENUSE"
20  PRINT:PRINT
30  PRINT "ENTER THE LENGTHS OF THE LEGS."
40  INPUT A,B
50  IF A<=0 OR B<=0 THEN GOTO 110
60  C=SQR(A↑2+B↑2)
70  N=C+.005:RC=INT(100*N)/100
80  PRINT "THE LENGTH OF THE HYPOTENUSE"
90  PRINT "IS ABOUT";RC
100 GOTO 20
110 PRINT "ENTER ONLY POSITIVE NUMBERS."
120 GOTO 20
130 END
```

Lesson: 47

AIM: How can we extend the Pythagorean relationship to determine if a triangle is acute or obtuse?

PERFORMANCE OBJECTIVES: The student will be able to...

1. explain a procedure with which one can determine if a triangle is acute or obtuse;
2. determine if a triangle is obtuse or acute.

VOCABULARY: acute triangle, obtuse triangle, right triangle

MOTIVATION:

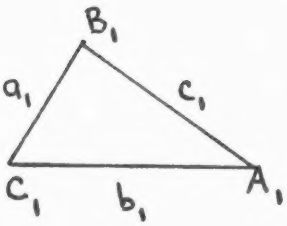
Can each of the following triples be the lengths of the sides of a right triangle?

1) 7, 24, 25
2) 9, 40, 41
3) 6, 8, 12

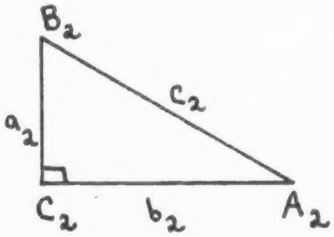
DEVELOPMENT:

1. In response to the motivation question, elicit that in a right triangle, $c^2 = a^2+b^2$ and that if $c^2 = a^2+b^2$, then a triangle is a right triangle. In example 1 and 2 above, $25^2 = 7^2+24^2$ and $41^2 = 9^2+40^2$; so these can be the lengths of the sides of right triangles. In the third, however, $12^2 \neq 6^2+8^2$.
2. Ask, "If $c^2 \neq a^2+b^2$, what kind of triangle can we have?" Elicit that the triangle might be either acute or obtuse. Ask, "How might we determine which type we have?"
3. Ask the students to consider the analysis which follows.

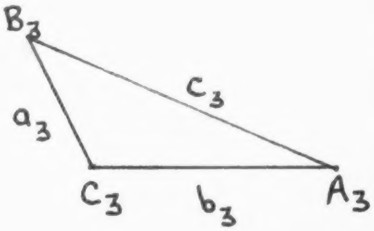
Triangle 1



Triangle 2



Triangle 3



4. In each triangle, refer to the length of $\overline{AB}$ as c, the length of $\overline{AC}$ as b, and the length $\overline{BC}$ as a. In each case, a = 4 units and b = 3 units. Notice that C_1 is an acute angle, C_2 is a right angle and C_3 an obtuse angle. Ask the students to measure $\overline{AB}$ in each triangle. Ask what happens to c as C_1 expands from acute to right to obtuse. Elicit that c gets larger and $\overline{AB}$ gets longer. Recall that in the right triangle, $c^2 = a^2+b^2$. Ask what would be true concerning c^2, a^2 and b^2 if angle C was acute or obtuse instead of right. Elicit that if $c^2 < a^2+b^2$, then the triangle is acute and if $c^2 > a^2+b^2$, then the triangle is obtuse. Remind the students that in this analysis, c must be the length of the longest side in the triangle.

5. Discuss problem 3 (motivation); since $12^2 > 6^2+8^2$ $(144>36+64)$, the triangle whose sides have lengths of 6 units, 8 units, and 12 units must be an obtuse triangle.

6. Complete the chart which follows with the students.

   $a^2+b^2 > c^2$ → ACUTE TRIANGLE
   $a^2+b^2 = c^2$ → RIGHT TRIANGLE
   $a^2+b^2 < c^2$ → OBTUSE TRIANGLE

APPLICATIONS:

1. Determine the type of triangle (acute, obtuse, or right) which each of the following triples of lengths can form.
   a) (7, 8, 12)
   b) (4, 5, 6)
   c) (5, 8, 11)
   d) (12, 16, 20)
   e) (5, 12, 14)
   f) (10 ,12, 13)

2. If two sides of a triangle measure 5 inches and 12 inches, how long would the third side have to be so that the triangle would be obtuse?

SUMMARY:

1. How must the lengths of the sides of a triangle be related so that the triangle is acute?

2. How must the lengths of the sides of a triangle be related so that the triangle is obtuse?

HOMEWORK:

1. State whether the given triples of lengths can form an acute, obtuse, or right triangle.
   a) (7, 8, 10)
   b) (7, 21, 25)
   c) (18, 24, 30)
   d) (4.5, 6, 7.5)
   e) (10, 11, 14)
   f) (6, 8, 11)
2. In triangle ABC, AB=8 and BC=15. If $\overline{AC}$ is the longest side of the triangle, what value(s) of AC will result in...
   a) an acute triangle.
   b) an obtuse triangle.
   c) a right triangle.
3. In triangle LMN, the longest side, $\overline{LM}$, has length 25 and $\overline{MN}$ has length 24. What length(s) for $\overline{LN}$ will result in a triangle that is...
   a) obtuse?
   b) right?
   c) acute?

HOMEWORK SOLUTIONS:

1a) $7^2+8^2=49+64=113>10^2$; therefore, an acute triangle.
b) $7^2+21^2=49+441=490<25^2$; therefore, an obtuse triangle.
c) $18^2+24^2=324+576=900=30^2$; therefore, a right triangle.
d) $4.5^2+6^2=20.25+36=56.25=7.5^2$; therefore, a right triangle.
e) $10^2+11^2=100+121=221>14^2$; therefore, an acute triangle.
f) $6^2+8^2=36+64=100<11^2$; therefore, an obtuse triangle.

2. Since $\overline{AC}$ is the longest side, if $(AB)^2+(BC)^2=(AC)^2$, we have a right triangle. $8^2+15^2=64+225=289=17^2$. Therefore if...
   a) $AC<17$, we have an acute triangle.
   b) $AC>17$, we have an obtuse triangle.
   c) $AC=17$, we have a right triangle.

3. If $(MN)^2+(LN)^2=(LM)^2$ we have a right triangle.
$(24)^2+x^2=25^2$ results in $576+x^2=625$ or $x^2=49$ or $x=7$.
   a) If $x<7$, then $(MN)^2+(LN)^2<(LM)^2$ and an obtuse triangle results.
   b) If $x=7$, then $(MN)^2+(LN)^2=(LM)^2$ and a right triangle results.
   c) If $x>7$, then $(MN)^2+(LN)^2>(LM)^2$ and an acute triangle results.

Lesson: 48

AIM: How do we develop a plan for a computer solution to determine whether a triangle is acute, right or obtuse when given the lengths of the sides?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop a plan for a computer solution to determine whether a triangle is acute, right or obtuse when given the lengths of the sides;
2. prepare a flowchart using the plan;
3. trace an algorithm for interchanging the contents of two memory cells.

VOCABULARY: obtuse triangle, right triangle, acute triangle, interchange the contents of two memory cells

MOTIVATION:

What would be the essential elements of a program to have the computer determine if a triangle is acute, right or obtuse when given the lengths of its sides?

DEVELOPMENT:

1. Elicit from the students that the lengths of the sides must be input (A,B,C) and that if...
   $A^2+B^2 > C^2$, we have an acute triangle,
   $A^2+B^2 = C^2$, we have a right triangle, and
   $A^2+B^2 < C^2$, we have obtuse triangle.
2. Ask the students to consider the outcome if the input statement was

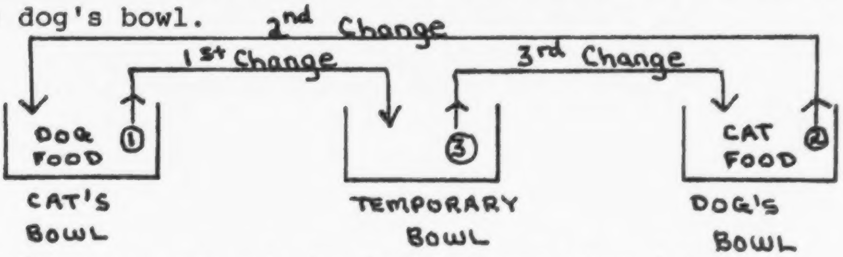
```
10  INPUT A,B,C
```

   and the response, while running the program was

```
RUN
? 3,5,4
```

   Note that this assigns 3 to variable A, 5 to variable B, and 4 to variable C. Ask whether A^2+B^2 > or = or < C^2. Elicit that $A^2+B^2=3^2+5^2=9+25=34>16=4^2$ and that the computer would decide that the triangle is acute. In fact, since $3^2+4^2 = 5^2$, the triangle is right.

3. Ask how this difficulty can be overcome. Elicit that either the largest value must be assigned to C or that the program should check for the largest value and interchange that value with the value of C if necessary. Relate the story of the pet owner who mistakenly placed dog food in the cat's bowl and cat food in the dog's bowl. Ask, "How can this mistake be corrected?" If need be, say to the students that a temporary bowl could be used. The dog food could be placed in the temporary bowl, the cat food then placed in its proper bowl, and the dog food taken out of the temporary bowl and put into the dog's bowl.



Consider a trace of the interchange algorithm.

	CAT'S BOWL	DOG'S BOWL	TEMPORARY BOWL
START	DOG FOOD	CAT FOOD	EMPTY
MOVE THE DOG FOOD TO THE TEMPORARY BOWL.	EMPTY	CAT FOOD	DOG FOOD
MOVE THE CAT FOOD TO THE CAT'S BOWL.	CAT FOOD	EMPTY	DOG FOOD
MOVE THE DOG FOOD TO THE DOG'S BOWL.	CAT FOOD	DOG FOOD	EMPTY

4. Elicit a plan for a program to have the computer classify a triangle as acute, obtuse or right when given the lengths of its sides.

Plan

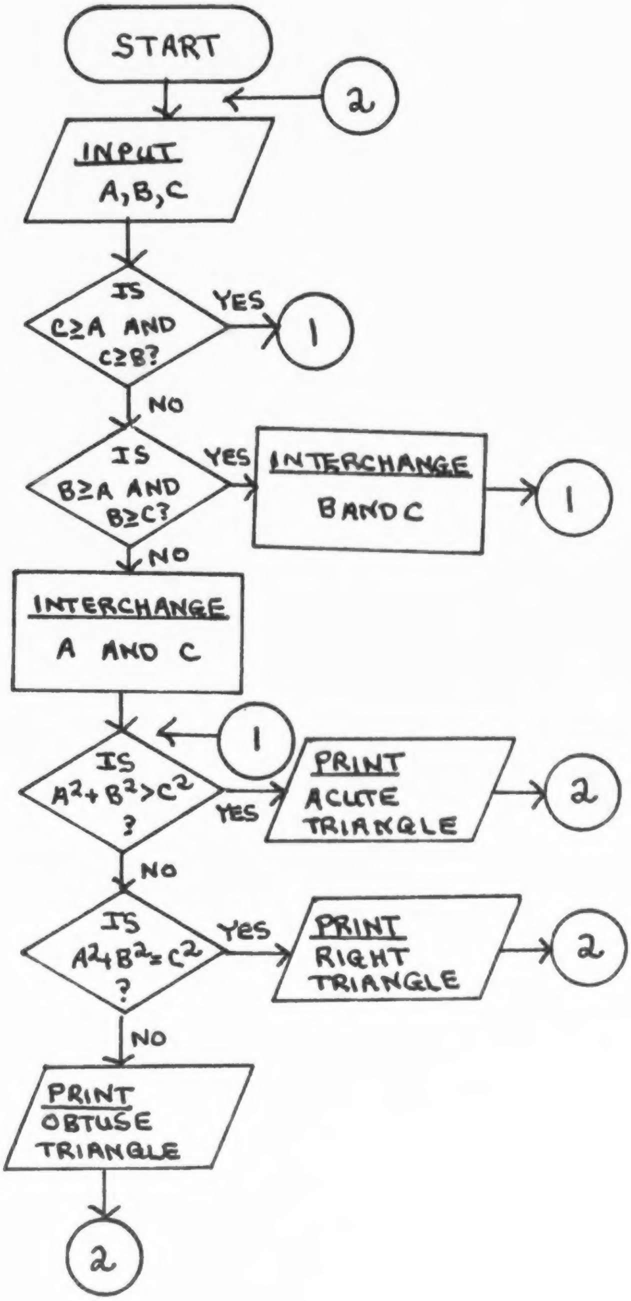
1) Input the lengths of the sides.
2) Determine which memory cell has the value of the longest side.
3) Interchange, if necessary.
4) Compare A^2+B^2 to C^2.

5) Print the appropriate message.

APPLICATIONS:

1. Ask the students to generate the flowchart which follows.

Flowchart


START
2
INPUT
A,B,C
IS
C≥A AND
C≥B?
YES
1
NO
IS
B≥A AND
B≥C?
YES
INTERCHANGE
B AND C
1
NO
INTERCHANGE
A AND C
1
IS
A²+B²>C²
?
YES
PRINT
ACUTE
TRIANGLE
2
NO
IS
A²+B²=C²
?
YES
PRINT
RIGHT
TRIANGLE
2
NO
PRINT
OBTUSE
TRIANGLE
2

2. Trace these statements by filling in the parentheses.

		A	B	T
	START	3	4	0*
a.	Give T the value of A.	( )	( )	( )
b.	Give A the value of B.	( )	( )	( )
c.	Give B the value of C.	( )	( )	( )

* Ask the students to note that previously unassigned memory cells contain zero.

SUMMARY:

1. When given the lengths of the sides, what are the components of a program to determine if a triangle is obtuse, acute or right?
2. Explain how we can interchange the contents of two memory cells.
3. How is interchanging memory cells different from interchanging the dog and cat food? (Elicit that the temporary memory cell does not become empty.)

HOMEWORK:

1. Trace the flowchart to determine the type of triangle (acute, obtuse or right) with A=10, B=6, and C=8 as the lengths of the sides.
2. Repeat question 1 with A=4, B=6, and C=5.
3. Indicate the statements and trace the values of the variables in an algorithm for interchanging the contents of memory cells X and Y. (Start with X=4 and Y=8.)

HOMEWORK SOLUTIONS:

1a) The variables' values are input:

A	B	C.
10	6	8

b) Is $8 \geq 10$ and $8 \geq 6$? (no)

c) Is $6 \geq 10$ and $6 \geq 8$? (no)

d) A and C are interchanged:

A	B	C
8	6	10

e) Is $8^2+6^2>10^2$? (no)

f) Is $8^2+6^2=10^2$? (yes)

g) RIGHT TRIANGLE is displayed.

2a) The variables are input:

A	B	C
4	6	5

b) Is $5 \geq 4$ and $5 \geq 6$? (no)

c) Is $6 \geq 4$ and $6 \geq 5$? (yes)

d) B and C are interchanged:

A	B	C
4	5	6

e) Is $4^2+5^2>6^2$? (yes)

f) ACUTE TRIANGLE is displayed.

3.

	X	Y	T
Start	4	8	0
Give T the value of X.	4	8	4
Give X the value of Y.	8	8	4
Give Y the value of T.	8	4	4

Lesson: 49L

AIM: How do we code a program to classify triangles by angles when given the lengths of their sides?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to classify triangles by angles when given the lengths of the sides;
2. enter, run and correct his/her program.

VOCABULARY: acute triangle, obtuse triangle, right triangle, interchange

MOTIVATION:

Write a program to classify triangles using the output shown.

Planned Output

```
ENTER THE LENGTHS OF THE SIDES.
? 5,4,3
THE TRIANGLE IS RIGHT.
ENTER THE LENGTHS OF THE SIDES.
?
```

DEVELOPMENT:

1. Ask the students to use yesterday's flowchart as a guide in writing the program up to the part involving the interchange of the contents of memory cells. Elicit the program segment which follows.

```
10  PRINT "ENTER THE LENGTHS OF THE SIDES."
20  INPUT A,B,C
30  IF C>=A AND C>=B THEN GOTO ___
40  IF B>=A and B>=C THEN GOTO ___
```

2. Ask, "We want the computer to interchange the contents of memory cells A and C. How can we code the interchange algorithm in BASIC?" Elicit the algorithm which follows.

IDEA	BASIC CODE
Give T the value of A.	T = A
Give A the value of C.	A = C
Give C the value of T.	C = T

3a) Ask the students to complete the program. Circulate around the room to check on the students' progress.

```
50  T = A
60  A = C
70  C = T
```

3b) Note that at this point line 30 can be completed to state: ...THEN GOTO 80.

```
80  IF A*A+B*B>C*C THEN GOTO ___
90  IF A*A+B*B=C*C THEN GOTO ___
100 PRINT "THE TRIANGLE IS OBTUSE."
110 GOTO 10
```

3c) Note that now we may complete line 40 to state: ...THEN GOTO 120.

```
120 T = B
130 B = C
140 C = T
150 GOTO 80
```

3d) Line 80 can be completed to state: ...THEN GOTO 160.

```
160 PRINT "THE TRIANGLE IS ACUTE."
170 GOTO 10
180 PRINT "THE TRIANGLE IS RIGHT."
190 GOTO 10
200 END
```

Line 90 should be completed to state: ...THEN GOTO 180.

4. As students complete the coding, they may enter and run their programs.

APPLICATIONS:

1. Allow the students to correct the errors in their programs on the computer.

2. Modify the program to produce the following output.

Planned Output

```
  CLASSIFYING TRIANGLES
  (Skip 2 lines.)
* THE SIDES 5, 6, AND 7
  FORM AN ACUTE TRIANGLE.
  (Skip 2 lines.)
  THE SIDES 16, 12, AND 9
  FORM AN OBTUSE TRIANGLE.
  (Skip 2 lines.)
  THE SIDES 8, 17, AND 15
  FORM A RIGHT TRIANGLE.
```

* Ask the students to note that the data is not input; a READ/DATA statement is used.

SUMMARY:

1. How do we code a segment of a program in BASIC designed to interchange the contents of memory cells R and P?
2. Name three different integers that can form the sides of a(n) obtuse triangle...right triangle...acute triangle.

HOMEWORK:

1. Code a segment of a program designed to interchange the contents of memory cells M and B.
2. Modify today's program so that it will produce the following output.

Planned Output

```
CLASSIFYING TRIANGLES BY ANGLES
(Skip 1 line.)
WHAT ARE THE LENGTHS OF THE SIDES? 7,24,26
THE TRIANGLE IS OBTUSE.
(Skip 1 line.)
WHAT ARE THE LENGTHS OF THE SIDES?
```

HOMEWORK SOLUTIONS:

1.
```
110 T = M
120 M = B
130 B = T
```

2. Add or change the following lines.

```
5   PRINT "CLASSIFYING TRIANGLES BY ANGLES"
7   PRINT
10  INPUT "WHAT ARE THE LENGTHS OF THE SIDES";A,B,C
```

Line 20 is omitted; lines 30 through 200 are the same, <u>except</u> lines 110, 170, 190 should be GOTO 6 instead of GOTO 10.

Lesson: 50

AIM: How do we find the lateral area of a right circular cylinder?
(You may permit the use of calculators in this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to...

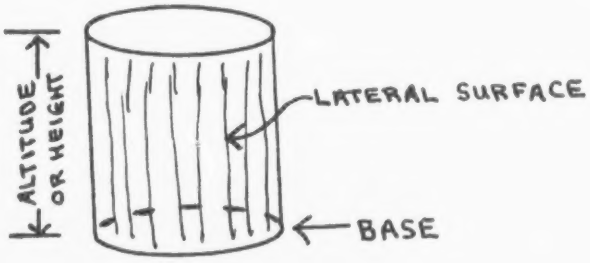
1. identify the altitude, base and lateral surface of a right circular cylinder;
2. find the lateral area of a right circular cylinder.

VOCABULARY: right circular cylinder, altitude, base, lateral area, height

MOTIVATION:

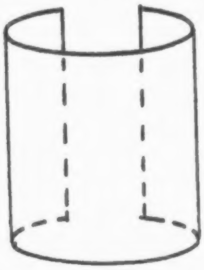
In ceramics class, Beth made a vase in which the edge is perpendicular to a circular base. How may square inches of wrapping paper are needed to completely cover the side of the vase if it is 10 inches high and the base has a 2 inch radius?

DEVELOPMENT:



1. Define the parts of a right circular cylinder as indicated in the diagram above. Ask the students to note that if the base is circular and the lateral surface is perpendicular to the base, then the shape is a right circular cylinder.
2. Ask "What kind of quantity is measured in square inches?" Elicit that the amount of wrapping paper needed to cover the lateral surface is called the lateral area. Ask what shape the wrapping paper must have in order to cover the cylinder.

Use a model cylinder made of cardboard; e.g., the insert in a roll of paper towels. Cut the cylinder and flatten it to show that the lateral surface is a rectangle



and elicit that the area of a rectangle is BASE x HEIGHT.

3. Ask "What part of the cylinder corresponds to the height of the rectangle? What part of the cylinder corresponds to the base of the rectangle?" Elicit that the altitude of the cylinder corresponds to the height of the rectangle and that the circumference of the cylinder's base corresponds to the base of the rectangle. Ask the students for the formula for the lateral area of a right circular cylinder using the rectangle as a model. Elicit that

   Lateral Area = Base x Height (in the rectangle).
   = $2\pi r$ x altitude (in the cylinder).

   If we represent the radius of the base as r, the altitude as h, and approximate π with 3.14, the formula becomes

   Lateral Area = $2(3.14)rh$.

4. Ask the students to find the lateral area of Beth's cylinder.

   Lateral area = $2(3.14)(2)(10) = 125.6$ sq. in.

5. Ask the students to complete the Applications problems.

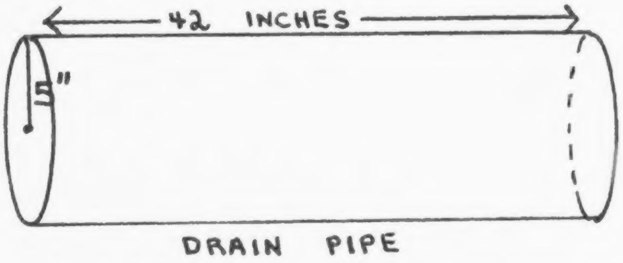
APPLICATIONS:

1. Find the lateral area of a right circular cylinder with a base radius of 3 inches and an altitude of 8 inches.

2. Find the lateral area of a right circular cylinder with an altitude of 12.5 cm and a base with a radius of 1.5 cm.

3. Find the lateral area of a right circular cylinder with a base diameter of 4" and an altitude of 14".

4. Find the lateral area of the right circular cylinder shown below.



SUMMARY:

How can we find the lateral area of a right circular cylinder?

HOMEWORK:

1. A right circular cylinder has an altitude of 16". The circumference of the base is 6.125". Find the lateral area.
2. Find the lateral area of a right circular cylinder with an altitude 14" high and a base radius of 5.5".
3. A tin can has an altitude of 6 inches and a base with a 4 inch diameter. Find the number of square inches in the lateral surface.
4. A copper tube with a radius of 1/2 inch has a length of 15 inches. Find the lateral area of the tube.
5. Translate the formula for the lateral surface area of a right circular cylinder into a BASIC statement.

HOMEWORK SOLUTIONS:

1. Circumference$\cdot$h = (6.125)(16) = 98 sq. in.
2. 2(3.14)(5.5)(14) = 483.56 sq. in.
3. $2(3.14)(\frac{4}{2})(6) = 75.36$ sq. in.
4. $2(3.14)(\frac{1}{2})(15) = 47.10$ sq. in.
5. LA = 2*3.14*R*H

Lesson: 51

AIM: How do we find the lateral area of a regular circular cone? (You may permit the use of calculators in this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to...

1. identify the slant height, base and lateral surface of a regular circular cone;
2. find the lateral area of a regular circular cone.

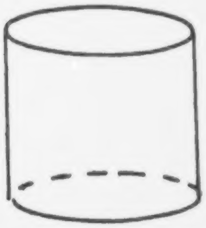
VOCABULARY: slant height, regular circular cone, apex, perpendicular

MOTIVATION:

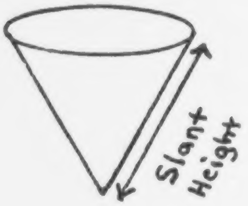
How much paper is needed to wrap around the cracker part of an ice cream cone if the circular base has a diameter of 2 inches and the distance from the point opposite the base to the edge of that base is 4 inches?

DEVELOPMENT:

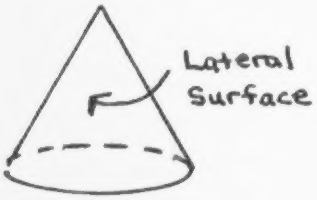
1. Ask the students to contrast a cylinder and a cone. Ask, "How do you describe the difference between a cylinder and a cone?" Elicit that the cylinder has two circular bases while the cone has one point (apex) and one circular base.
2. Draw a diagram of a cone, indicating its slant height (the distance from the apex to the edge of the base).



CYLINDER



CONE (BASE AT TOP)



CONE (BASE AT BOTTOM)

3. Elicit that the amount of paper needed to wrap around the cone is equal to the lateral area of the cone. The formula for the lateral area of a regular cone (with the

apex of the cone lying on an imaginery line perpendicular to the base and passing through the center of the base) is:

Lateral Area = $\frac{1}{2}$ c·s where
c = circumference of base and
s = slant height.

3. Ask how we can find the lateral area of the cone in the motivation question. Elicit that $C = \pi d$ or $C = 2\pi r$, and that the lateral area formula is either:

$$\text{LATERAL AREA} = \frac{1}{2}\pi ds \text{ or}$$

$$\text{LATERAL AREA} = \frac{1}{2}(2\pi r)s = \pi rs.$$

4. Using $\pi=3.14$ in the original problem,

$$\text{Lateral area} = \frac{1}{2}(3.14)(2)(4) = 12.56 \text{ sq. in.}$$

5. Ask the students to complete the Applications problems.

APPLICATIONS:

Use $\pi=3.14$ for problems 1-4.

1. Find the lateral area of a cone with a circumference of 6.25" at the base, and a slant height of 4".
2. A funnel, in the shape of a cone, has a base with diameter 6" and a slant height of 8". Find its lateral area.
3. A mound of sand, shaped like a cone, is to be covered by a special material. How many square inches of material are needed if the cone has a slant height that is 4 ft. long and has a base with a 4 foot diameter?
4. Find the lateral area of the cones with dimensions listed below.

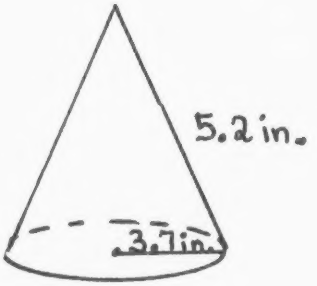
	BASE RADII	BASE DIAMETERS	SLANT HEIGHTS
a)	---	4.5	8
b)	3.2	---	6.25
c)	18	---	3.5

SUMMARY:

1) What is the formula for the lateral area of a cone?
2) Which is longer, the height or the slant height of a cone?

HOMEWORK:

1. Find the lateral area of the cones with the following dimensions. Use $\pi=3.14$.
   a) Circumference = 40.25 cm, slant height = 8 cm.
   b) Diameter of base = 12 in., slant height = $9\frac{1}{2}$ in.
   c) Radius of base = 4.7 in., slant height = 11 in.
2. Find the lateral area of the cone drawn below. Use $\pi=3.14$.



3. A hat in the shape of a cone is used as a Halloween decoration. Find the lateral area if the slant height is 15 inches and the diameter of the base is 8 inches. Use $\pi=3.14$.
4. Translate the formula $LA = \frac{1}{2}(3.14)ds$ into the BASIC language.

HOMEWORK SOLUTIONS:

1a) $LA = \frac{1}{2}c \cdot s = \frac{1}{2}(40.25)(8) = 161$ sq. cm
b) $LA = \frac{1}{2}\pi ds = \frac{1}{2}(3.14)(12)(9.5) = 178.98$ sq. in.
c) $LA = \pi rs = 3.14(4.7)(11) = 162.338$ sq. in.

2. $LA = \pi rs = (3.14)(3.7)(5.2) = 60.4136$ sq. in.

3. $LA = \frac{1}{2}\pi ds = \frac{1}{2}(3.14)(8)(15) = 188.40$ sq. in.

4. LA = .5*3.14*D*S or LA = 3.14*D*S/2

Lesson: 52

AIM: How do we find the total surface area of a regular circular cone and a right circular cylinder?
(The students may use calculators for this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to...

1. define the total surface area of a regular circular cone;
2. define the total surface area of a right circular cylinder;
3. find the total surface area of a regular circular cone;
4. find the total surface area of a right circular cylinder.

VOCABULARY: total surface area

MOTIVATION:

A space capsule, in the shape of a regular circular cone, has a slant height of 30 feet and a radius (at the base) of 10 feet. How many square feet of special ceramic tile is needed to insulate the capsule from the high temperatures of reentry into our atmosphere?

DEVELOPMENT:

1. Ask "How can we use the formula for lateral area to find the solution to the motivation problem?" Elicit that lateral area is only part of the answer. The base of the capsule must be insulated also. Ask, "How can we find the total surface area?"
2. Elicit that the total surface area is the sum of the lateral area and the area of the base (or bases, in a cylinder).
   Furthermore, each base is circular, with an area equal to πr^2.
3. Ask, "What is the formula for total surface area of a regular circular cone?" Elicit $SA = \pi rs+\pi r^2$.
4. Repeat to elicit the formula for a right circular cylinder: $SA = 2\pi rh + 2\pi r^2$.

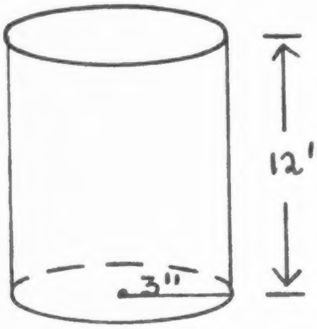
5. Ask the students to compute the answer for the space capsule question using $\pi=3.14$.

$SA = \pi rs+\pi r^2$

$SA = 3.14(10)(30)+3.14(10)^2$

$SA = 1256$ sq. ft.

6. Find the total surface area of the cylinder shown below using $\pi=3.14$.



$SA = 2\pi rh+2\pi r^2$

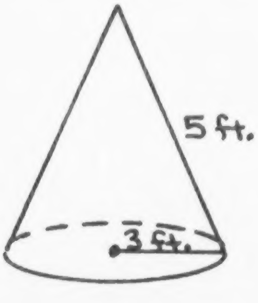
$SA = 2(3.14)(3)(12)+2(3.14)(3)^2$

$SA = 282.60$ sq. in.

APPLICATIONS:

In 1-4, use $\pi=3.14$.

1. Find the total surface area for each regular circular cone.
   a) radius = 4.3, slant height = 3.2
   b) diameter = 16.2, slant height = 10
2. Find the total surface area for each right circular cylinder.
   a) radius = 4, altitude = 7.5
   b) diameter = 8, altitude = 4.1
3. Find the amount of tin used in manufacturing a can with a base radius of 3.2" and an altitude of 8.1".
4. An Indian teepee is shaped like a regular circular cone. Find the number of square feet needed to cover the teepee (including the floor).



SUMMARY:

How can we determine the total surface area of a regular cone and of a right circular cylinder?

HOMEWORK:

1. Find the total surface area for the right circular cylinders with dimensions given below. Use $\pi=3.14$.
   a) base radius = 3.4 cm, altitude = 8.5 cm
   b) base radius = 2.1 in., altitude = 6.2 in.
2. Find the total surface area for a regular circular cone with a base radius = 7.5" and slant height = 8.3". Use $\pi=3.14$.
3. A hatbox, in the shape of a cylinder, is 8.8" high, with a base radius of 6.25". Using $\pi=3.14$, how many square inches of wrapping paper is needed for the hatbox?
4. Use the dimensions in example 1a to show that $SA = 2\pi r(h+r)$ is equivalent to $SA = 2\pi rh+2\pi r^2$.

HOMEWORK SOLUTIONS:

1a) $SA = 2\pi rh + 2\pi r^2$
$SA = 2(3.14)(3.4)(8.5) + 2(3.14)(3.4)^2$
$SA = 254.0888$ sq. cm

1b) $SA = 2\pi rh + 2\pi r^2$
$SA = 2(3.14)(2.1)(6.2) + 2(3.14)(2.1)^2$
$SA = 109.4604$ sq. in.

2. $SA = \pi rs + \pi r^2$
$SA = 3.14(7.5)(8.3) + 3.14(7.5)^2$
$SA = 372.09$ sq. in.

3. $SA = 2\pi rh + 2\pi r^2$
$SA = 2(3.14)(6.25)(8.8) + 2(3.14)(6.25)^2$
$SA = 590.7125$ sq. in.

4. $SA = 2\pi r(h+r)$
$SA = 2(3.14)(3.4)(8.5+3.4)$
$SA = 2(3.14)(3.4)(11.9)$
$SA = 254.0888$ sq. cm

Lesson: 53

AIM: How do we find the volume of a regular circular cone and a right circular cylinder?
(The students may use calculators for this lesson.)

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the volume of a regular circular cone;
2. find the volume of a right circular cylinder.

VOCABULARY: volume, regular circular cone, right circular cylinder, solid figure

MOTIVATION:

In metal shop, Michael made a can in the shape of a right circular cylinder. The altitude and diameter of the base were both 6". Find the holding capacity of the can.

DEVELOPMENT:

(Note: 3.14 will be used as the approximation for π throughout this lesson.)

1. Review with the students that the holding capacity of a can is determined by the volume of the can. Review the formula for the volume of a rectangular prism, $V = L \cdot W \cdot H$, where L = length, W = width, and H = height or altitude.
2. Ask the students to write the formula as $V=(L \cdot W) \cdot H$. Ask, "In reference to the rectangular prism, what does the product $L \cdot W$ represent?" Elicit that this is the area of the base, and so V = (Area of base)(height).

3a. Ask the students to consider that the same principle might apply to cylinders (and all solids with constant cross-sectional area).

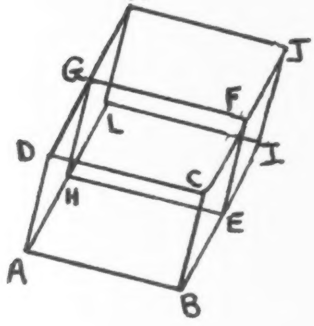


Fig. 1

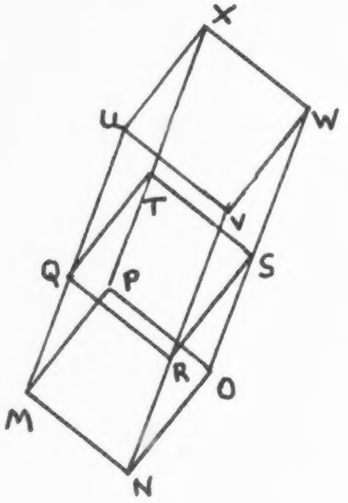


Fig. 2

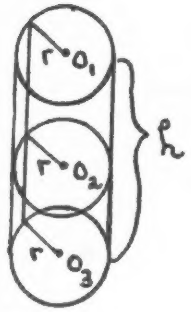


Fig. 3

If, in each of these solid figures, the cross-sectional areas or "slices" are equal, the volume equals the area of the base multiplied by the distance between the upper and lower bases. For example, in Fig. 1, if the areas of ABCD, EFGH, and IJKL are equal, V = (Area of ABCD)(BI). In Fig. 2, if the areas of NMOP, QRST, and UVWX are equal, V = (Area of MNOP)(MU). In Fig. 3, if the radii of circles O_1, O_2, and O_3 are equal in length, the areas of the circles are equal, and V = (Area of circle O_1)(h).

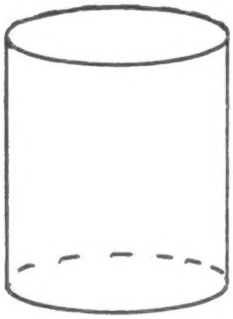
3b. Ask, "What should the formula be for the volume of a cylinder?" Elicit that a cylinder is illustrated by Fig. 3, and that V = (Area of base)(altitude) or $V = \pi r^2 h$.

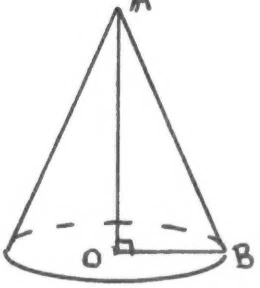
4. Ask the students to find the volume of Michael's shop project.

   $V = \pi r^2 h$

   $V = (3.14)(6)^2(6) = 678.24$ cubic inches.

5. State that the volume of a regular circular cone is given by $V = \frac{1}{3}\pi r^2 h$. Demonstrate this using a cone and a cylinder with equal bases and equal altitudes. Show that if the contents (e.g., sand) of the cone is poured into the cylinder, exactly three fills are needed.





Note that in the diagram shown, $\overline{OA}$ is the altitude (h) and $\overline{AB}$ is the slant height (s).

6. Ask the students to complete the Applications problems.

APPLICATIONS:

1. Find the volume of a cylinder with a base radius of 3.6 and an altitude of 8.
2. Find the volume of a cone with base radius 2.4 and altitude 6.9.
3. How many cubic inches of wax are needed to make a cylindrical candle with a 1 inch radius and a length of 12 inches?
4. How many cubic inches of liquid can be held in a cone-shaped container, with base radius 4.1 inches and altitude 6 inches?
5. A fish tank in the shape of a cylinder is 40 cm in height and has a 25 cm radius. Find the volume of the tank.
6. If 1000 cubic centimeters hold 1 liter, how many liters of water can be held by the tank in example 5.

SUMMARY:

What are the formulas for the volume of a right circular cylinder and the volume of a regular circular cone?

HOMEWORK:

1. Find the volume of each of the following regular cones or right circular cylinders.
   a) cylinder, base radius = 2.1 inches, altitude = 6 inches
   b) cone, base radius = 1.5 inches, altitude = 8 inches
   c) cylinder, base diameter = 12 cm, altitude = 4.5 cm
2. A right circular cylinder, with a base radius of 5 feet and a height of 7.2 feet is used to hold water. Find the capacity of the cylinder in cubic feet.
3. If 1 cubic foot holds approximately 7.5 gallons, determine the capacity of the cylinder from example 2 in gallons.
4. A rock pile is cone-shaped, with a base diameter of 8 feet and an altitude of 3.75 feet. What volume of rocks does it contain?
5. If the altitude of a cylinder is doubled, what would be the effect on the volume? ...what if the radius is doubled instead?

HOMEWORK SOLUTIONS:

1a) $V = \pi r^2 h = (3.14)(2.1)^2(6) = 83.0844$ cu. in.

b) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}(3.14)(1.5)^2(8) = 18.84$ cu. in.

c) $V = \pi r^2 h = (3.14)(6)^2(4.5) = 508.68$ cc

2. $V = \pi r^2 h = (3.14)(5)^2(7.2) = 565.2$ cu. ft.

3. 1 cu. ft. = 7.5 gal.
562.2 cu. ft. = 7.5(565.2) = 4239 gal.

4. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}(3.14)(4)^2(3.75)$
$= 62.8$ cu. ft.

5. If the altitude doubled, the volume would also double, but if the radius doubled instead, the volume would quadruple. For e.g., if

altitude = 1 and radius = 1, $V=(3.14)(1)^2(1)=3.14$

and if altitude = 2 and radius = 1, $V=(3.14)(1)^2(2)=6.28$;

$6.28=2(3.14)$.

On the other hand, if altitude = 1 and radius = 2,

$V=(3.14)(2)^2(1)=12.56$;

$12.56=4(3.14)$.

Lesson: 54L

AIM: How do we write a program to find the total surface area of a right circular cylinder?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop a plan for a program to find the total surface area of a right circular cylinder;
2. prepare a flowchart using the plan;
3. write a program to find the total surface area;
4. enter, run, and correct his/her program.

VOCABULARY: total surface area, right circular cylinder, altitude, radius

MOTIVATION:

The packaging department of a large food processor must anticipate the amount of aluminum used by their canned food products. With many different sized cans, how can the department quickly find the surface area used by the various cans?

DEVELOPMENT:

(Note: 3.14 will be used as the approximation for π throughout this lesson.)

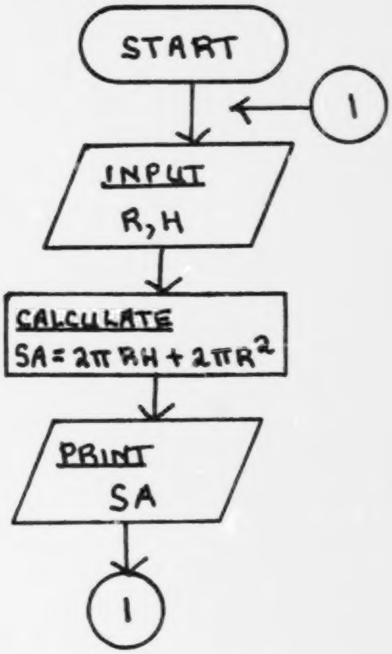
1. Elicit that a computer program would be most helpful. Ask the students for the formula for total surface area of a cylinder and to develop a plan for a computer program. The plan may evolve as follows.

   Plan

   1) Enter the radius (R) and altitude (H).
   2) Compute the total surface area: $SA = 2\pi RH+2\pi R^2$.
   3) Print the results.
   4) Go to step 1.

2. Ask the students to prepare a flowchart, planned output, and write the program. One possibility follows.

Flowchart



Planned Output

```
ENTER THE RADIUS AND THE ALTITUDE.
? 5,18
THE TOTAL SURFACE AREA IS
722.2 SQ. UNITS.
ENTER THE RADIUS AND THE ALTITUDE.
?
```

Program

```
10  PRINT "ENTER THE RADIUS AND THE ALTITUDE."
20  INPUT R,H
30  SA = 2*3.14*R*H+2*3.14*R*R
40  PRINT "THE TOTAL SURFACE AREA IS"
50  PRINT SA;"SQ. UNITS."
60  GOTO 10
70  END
```

3. Allow the students to enter, run and edit their programs. Circulate around the room to offer help where needed.
4. As the students complete the program, ask them to work on the Applications problems.

APPLICATIONS:

1. Modify the program to print a title such as the one which follows.

   Planned Output

   ```
   COMPUTING THE TOTAL SURFACE AREAS
   **OF RIGHT CIRCULAR CYLINDERS**
   (Skip 2 lines.)
   ENTER THE RADIUS AND THE ALTITUDE.
   ?
   ```

2. Modify the program to allow the values of R and H to be read in as data.
3. Change the program to include a statement which computes the cost of the cylinder if its dimensions are entered in inches and the surface material costs $.00035 per square inch.

SUMMARY:

What are the essential elements of a program which computes the total surface areas of right circular cylinders?

HOMEWORK:

1. Compute the total surface area of a right circular cylinder with a base radius of 4 inches and an altitude of 1 foot.
2. Develop a flowchart, design a planned output, and write a program to compute the volumes of right circular cylinders using READ and DATA. Use a flag to signal the end of the data. Use the following data: 2,3,7,4,3,8,0,0.
3. Develop a flowchart, design a planned output, and write a program to compute the volume of each of 2 cylinders and display which has the larger volume and how much larger.

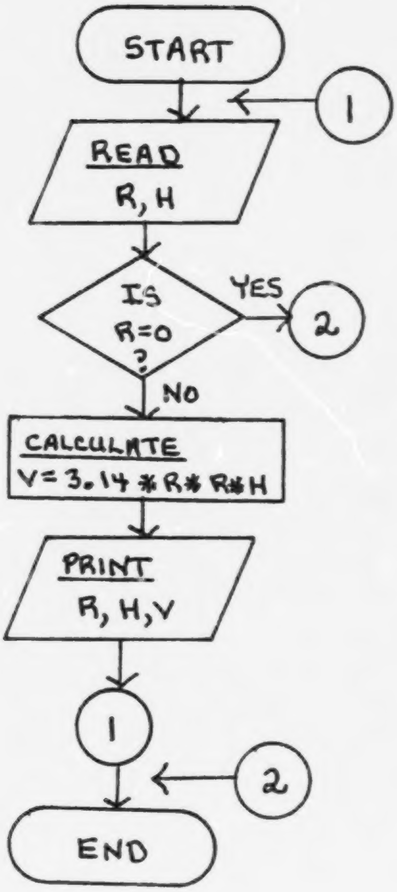
HOMEWORK SOLUTIONS:

1. $SA = 2\pi rh + 2\pi r^2$

   $SA = 2(3.14)(4)(12)+2(3.14)(4)^2$

   $SA = 401.92$ sq. in.

2. Flowchart


START
1
READ
R, H
IS
R=0
?
YES
2
NO
CALCULATE
V=3.14*R*R*H
PRINT
R, H, V
1
2
END

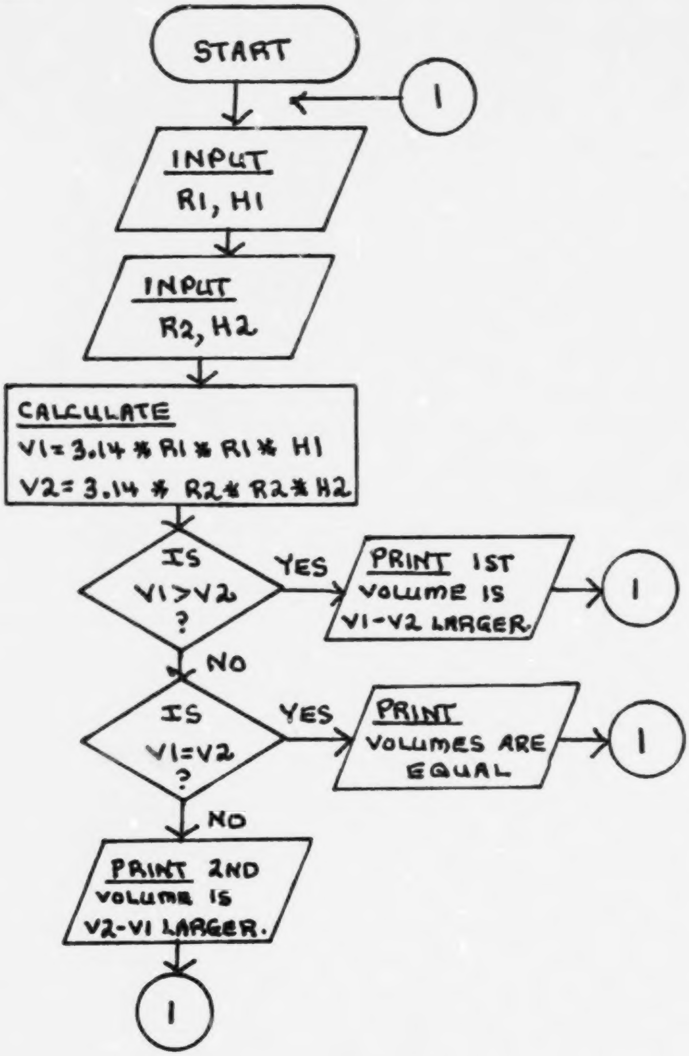
Planned Output

```
CYLINDER VOLUMES
(Skip 1 line.)
RADIUS = 2
ALTITUDE = 3
VOLUME = 37.68
(Skip 1 line.)
RADIUS = 7
ALTITUDE = 4
VOLUME = 615.44
(Skip 1 line.)
RADIUS = 3
ALTITUDE = 8
VOLUME = 226.08
```

Program

```
10  PRINT "CYLINDER VOLUMES"
20  PRINT
30  READ R,H
40  IF R=0 THEN GOTO 110
50  V = 3.14*R*R*H
60  PRINT "RADIUS =";R
70  PRINT "ALTITUDE =";H
80  PRINT "VOLUME =";V
90  GOTO 20
100 DATA 2,3,7,4,3,8,0,0
110 END
```

3. Flowchart


START
1
INPUT
R1, H1
INPUT
R2, H2
CALCULATE
V1=3.14*R1*R1*H1
V2=3.14*R2*R2*H2
IS
V1>V2
?
YES
PRINT 1ST
VOLUME IS
V1-V2 LARGER.
1
NO
IS
V1=V2
?
YES
PRINT
VOLUMES ARE
EQUAL
1
NO
PRINT 2ND
VOLUME IS
V2-V1 LARGER.
1

Planned Output

```
ENTER R1 AND H1.
? 2,3
ENTER R2 AND H2.
? 3,2
THE SECOND VOLUME IS LARGER
BY 18.84 CUBIC UNITS.
(Skip 1 line.)
ENTER R1 AND H1.
```

Program

```
10  PRINT "ENTER R1 AND H1."
20  INPUT R1,H1
30  PRINT "ENTER R2 and H2."
40  INPUT R2,H2
50  V1 = 3.14*R1*R1*H1
60  V2 = 3.14*R2*R2*H2
70  IF V1>V2 THEN GOTO 120
80  IF V1=V2 THEN GOTO 150
90  PRINT "THE SECOND VOLUME IS LARGER"
100 PRINT "BY";V2-V1;"CUBIC UNITS."
110 PRINT:GOTO 10
120 PRINT "THE FIRST VOLUME IS LARGER"
130 PRINT "BY";V1-V2;"CUBIC UNITS."
140 PRINT:GOTO 10
150 PRINT "THE VOLUMES ARE EQUAL."
160 PRINT:GOTO 10
170 END
```

Lesson: 55

AIM: How do we determine the probability of events taking place?

PERFORMANCE OBJECTIVES: The student will be able to...

1. distinguish between experimental (empirical) and theoretical probability;
2. determine the experimental (empirical) probability of an event happening;
3. define and determine the theoretical probability of an event happening.

VOCABULARY: experimental (empirical) probability, theoretical probability, outcome (event), sample space, accumulated, probability ratio, equally likely, biased, unbiased

MOTIVATION:

Divide the students into groups of four. Ask each group to conduct one of the three experiments described below. In addition to providing the groups with the recording sheets and the materials necessary for performing the experiment, provide them with calculators.

EXPERIMENT I

Materials: 4 dice

Instructions:

1. Each student in the group is to roll a die fifty times and record the number of times that a "2" is rolled.
2. The data is to be recorded on a recording sheet similar to the one which follows.

Die Experiment

Students' Names	Number of Times a "2" was Rolled	Number of Rolls	Accumulated Number of Times a "2" was Rolled A	Accumulated Number of Rolls B	Experimental Probability A/B
		50		50	
		50		100	
		50		150	
		50		200	

3. The group is then to compute the accumulated number of times that a "2" was rolled (A) and the ratio of the accumulated number of times that a "2" was rolled to the accumulated number of rolls (A/B) and express the ratio as a decimal rounded to 3 places.

EXPERIMENT II

Materials: 4 coins

Instructions:

1. Each student in the group is to toss a coin fifty times and record the number of times that a "head" is tossed.
2. The data is to be recorded on a recording sheet similar to the one which follows.

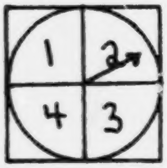
Coin Experiment

Students' Names	Number of Times a "head" was Tossed	Number of Tosses	Accumulated Number of Times a "head" was Tossed A	Accumulated Number of Tosses B	Experimental Probability A/B
		50		50	
		50		100	
		50		150	
		50		200	

3. The group is then to compute the accumulated number of times that a "head" was tossed (A) and the ratio of the accumulated number of times that a "head" was tossed to the accumulated number of tosses (A/B) and express the ratio as a decimal rounded to 3 places.

EXPERIMENT III

Materials: 4 spinners as pictured below



Instructions:

1. Each student in the group is to spin a spinner fifty times and record the number of times that a "3" appears.
2. The data is to be recorded on a recording sheet similar to the one which follows.

Spinner Experiment

Students' Names	Number of Times a "3" Appeared	Number of Spins	Accumulated Number of Times a "3" Appeared A	Accumulated Number of Spins B	Experimental Probability A/B
		50		50	
		50		100	
		50		150	
		50		200	

3. The group is then to compute the accumulated number of times that a "3" appeared (**A**) and the ratio of the accumulated number of times that a "3" appeared to the accumulatd number of spins (**A/B**) and express the ratio as a decimal rounded to 3 places.

(Note to the teacher: You may wish to model the completion of one of the tables before asking the entire class to proceed with the experiment.)

DEVELOPMENT:

1. Say to the students that today we will be predicting and assigning a number to the likelihood or probability that certain events or outcomes will occur. The experiments we just conducted will provide us with evidence that our predictions are correct.

2. Directing the students' attention to Experiment I, ask, "What is the probability that a '2' will be rolled on a fair die?" Elicit that since there are six possible, equally likely outcomes 1, 2, 3, 4, 5, 6 and "2" is one of these outcomes, the probability ratio is 1/6. Write the following definitions on the chalkboard.

   Definitions:

   An outcome (event) is the result of some activity or experiment.
   A sample space is a set of all possible outcomes for the activity or experiment.
   The theoretical probability of an event (for fair, unbiased objects) is the number of ways that an event can occur, divided by the total number of possibilities.

   Ask, "What would be considered an unfair or biased object?" Elicit some examples...tack, paper cup, "weighted" coin, "loaded" die. Ask the students to change 1/6 to a decimal rounded to three places. Ask, "Did the experiment provide evidence that the probability assigned, approximately .167, is correct?" Ask the students to notice that the experimental (empirical) probability comes closer to the theoretical probability as the accumulated number of rolls increases.

3. Directing the students' attention to Experiment II, ask, "What is the sample space? What is the probability that a 'head' is tossed on a fair coin?" Ask the students to change 1/2 to a three-place decimal. Ask, "Did the experiment provide evidence that the probability assigned, .500, is correct?"

4. Directing the students' attention to Experiment III, ask, "What is the sample space? What is the probability that a '3' appears when we spin the spinner?" Ask the students to change 1/4 to a three-place decimal. Ask, "Did the experiment provide evidence that the probability assigned, .250, is correct?"

5. Say to the students that the probability of an event occurring is always represented by a number from zero through one. Then ask the students to complete the Applications problems.

APPLICATIONS:

What is the probability ...

1. that the sun will rise tomorrow?
2. that the Empire State Building will fly away?
3. of drawing a king of hearts from a standard deck of cards?
4. of guessing a person's birthday in the month of June?
5. of drawing a white ball from a bowl with all white balls?
6. of rolling a number that is less than five in a single roll of a fair die?
7. of the principal of this school being an all-American fullback?
8. of tossing a "tail" in a single toss of a fair coin?
9. that the first digit on a New York State license plate will be a "3"?
10. that the last letter on a New York State license plate will be a "Q"?

SUMMARY:

1. What is the probability of an impossible event?
2. What is the probability that an event is certain to occur?
3. If an event can occur in two ways out of five equally likely possibilities, what number represents the probability of the event occurring?
4. How does the probability ratio of tossing a "head" on a fair coin compare with the probability ratio of rolling an "even number" on a fair die?

HOMEWORK:

1. A letter is chosen at random from the word "COMPUTER." What is the probability that the letter will be a(n)...
    a) "T"?
    b) vowel?
    c) "A"?
2. A box contains 3 red marbles and 7 yellow marbles. If one marble is picked at random, what is the probability, expressed in decimal form, that it is...
    a) red?
    b) yellow?
    c) blue?
3. A student does not know the answer to a test question and takes a guess.
    a) Find the probability that the answer is correct if the question is...
        i) a multiple-choice question with four choices.
        ii) a true-false question.
        iii) a question where the choices given are "sometimes, always, or never."
    b) On which type of question does the student have the best chance of guessing correctly?
4. Lorraine has 2 quarters, 1 dime, and 3 nickels in her pocket. She pulls out a coin at random. What is the probability that the coin is worth...
    a) exactly 10 cents?
    b) exactly 5 cents?
    c) exactly 25 cents?
    d) exactly 50 cents?
    e) less than 25 cents?
    f) less than 50 cents?

5. A standard deck of 52 cards is shuffled and one card is drawn. What is the probability that the card is ...

   a) the king of diamonds?
   b) any king?
   c) any diamond?
   d) a red seven?
   e) a picture card (jack, king, queen)?
   f) an even number in any suit (spade, club, hearts, diamonds)?

HOMEWORK SOLUTIONS:

1a) 1/8 b) 3/8 c) 0

2a) .3 b) .7 c) 0

3a) i) 1/4 ii) 1/2 iii) 1/3

3b) The student has the best chance of quessing correctly if the type is a true-false question.

4a) 1/6 b) 3/6 or 1/2 c) 2/6 or 1/3
d) 0 e) 4/6 or 2/3 f) 1

5a) 1/52 b) 4/52 or 1/13 c) 13/52 or 1/4
d) 2/52 or 1/26 e) 12/52 or 3/13 f) 20/52 or 5/13

Lesson: 56L

AIM: How do we simulate an experiment to determine the probability of obtaining a prime number on a throw of a single die?

PERFORMANCE OBJECTIVES: The student will be able to...

1. use the random function in a BASIC program;
2. plan (flowchart) the simulation of a probability experiment involving the throwing of a single die;
3. design an output and code the program;
4. enter, run, and debug the program.

VOCABULARY: simulate, random function (RND(1)), prime number, sieve, equally likely

MOTIVATION: What is the probability of getting a prime number when throwing a single die?

DEVELOPMENT:

1. Elicit a response to the motivation problem. Say to the students that today they will be planning and writing a program that will simulate a probability experiment involving the throwing of a single die. Present the following problem for a computer solution.

   Problem: Have the computer simulate the throwing of a single die one thousand times. Have it then determine and display the experimental probability of getting a prime number.

2. Develop the analysis which follows with the students.

Analysis

Ask, "How are we going to simulate the throwing of a die?" Say to the students that, in order to give the effect of throwing a die, we want to generate one thousand random whole numbers from one through six. One of the computer's BASIC functions, whose code name is RND(1) on an Apple computer, will be useful here. (Note: On some computers RND(0) is used. Check your manual.) Let us see how it works. Ask the students to type in and run the program which follows.

Program

```
10 FOR T = 1 TO 5
20 D = RND(1)
30 PRINT D
40 NEXT T
50 END
```

Ask them to notice that five random numbers between 0 and 1 are produced every time the program is run. The random function gives us nine digit decimals between zero and one. We want whole numbers from one through six. Let us look at the smallest and largest random numbers that RND(1) produces, to see how we can get numbers from one through six.

ALGORITHM	SMALLEST RND(1)	LARGEST RND(1)
	.000000001	.999999999
i) Multiply: 6*RND(1)	.000000006	5.999999994
ii) Use INT: INT(6*RND(1))	0	5

INT(6*RND(1)) will give us random whole numbers from zero through five. Ask, "How do we get random whole numbers from one through six?" Elicit INT(6*RND(1))+1.

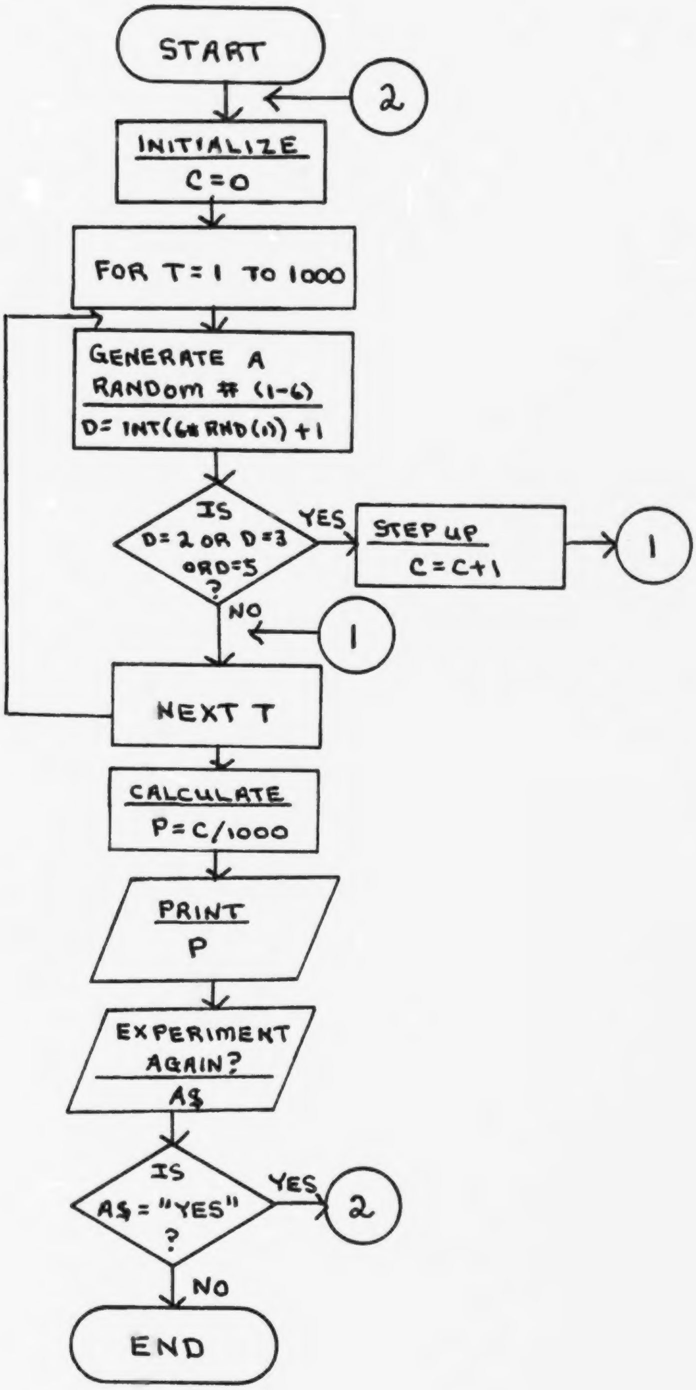
3. Ask the students to incorporate the algorithm into a plan as follows.

Plan

```
 1) Initialize the counter for the primes:  C=0.
 2) Simulate the one thousand throws:  FOR T=1 TO 1000.
 3) Simulate throwing a single die:  D=INT(6*RND(1))+1.
 4) Check:  Is D=2 or D=3 or D=5?
    a)  If yes, step up the count:  C=C+1. Then go to 5.
    b)  If no, continue.
 5) NEXT T
 6) Calculate the probability:  P=C/1000.
 7) Print P.
 8) Do the experiment again?  Answer yes or no:  A$.
 9) Check:  Is A$= "YES"?
    a)  If yes, go to 1.
    b)  If no, continue.
10) End.
```

4. Ask the students to develop a flowchart as follows.

Flowchart


START
2
INITIALIZE
C=0
FOR T=1 TO 1000
GENERATE A RANDOM # (1-6)
D= INT(6*RND(1)) +1
IS D=2 OR D=3 OR D=5 ?
YES
STEP UP
C=C+1
1
NO
1
NEXT T
CALCULATE
P=C/1000
PRINT
P
EXPERIMENT AGAIN?
A$
IS A$ = "YES" ?
YES
2
NO
END

5. Ask the students to design an output as follows.

Planned Output

```
THROWING A PRIME NUMBER
WITH A SINGLE DIE
(Skip 2 lines.)
THE PROBABILITY IS .502
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? YES
(Skip 2 lines.)
THE PROBABILITY IS .498
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? NO
(Skip 1 line.)
EXPERIMENTS CONCLUDED!
```

6. Ask the students to code the program which follows and run it several times.

Program

```
10  PRINT "THROWING A PRIME NUMBER"
20  PRINT "WITH A SINGLE DIE"
30  PRINT:PRINT
40  C=0
50  FOR T=1 TO 1000
60  D=INT(6*RND(1))+1
70  IF D=2 OR D = 3 OR D=5 THEN C=C+1
80  NEXT T
90  P=C/1000
100 PRINT "THE PROBABILITY IS";P
110 PRINT
120 PRINT "DO YOU WISH TO DO THE EXPERIMENT AGAIN?"
130 INPUT A$
140 IF AS = "YES" THEN 30
150 PRINT:PRINT "EXPERIMENTS CONCLUDED!"
160 END
```

7. Ask the students who complete the program to work on the Applications problems.

APPLICATIONS:

1. About 200 B.C., the Greek scholar Eratosthenes discovered a pattern for separating all counting numbers into two classes. One class contains the nonprime numbers and the other class contains the prime numbers. We will examine the scheme, called the Sieve of Eratosthenes, as it applies to the counting numbers from one through one hundred.

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	94
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

i) Cross out 1.
ii) Begin with 2; cross out all its multiples except itself.
iii) Find the next number not crossed out; it is 3. Cross out all its multiples except itself.
iv) Find the next number not crossed out; it is 5. Cross out all its multiples except itself.
v) Continue this process until you are all the way through the list.

The numerals that have not been crossed out are, of course, the names of numbers that are not multiples of two smaller counting numbers. These are the prime numbers.

a) List the set of prime numbers from one through one hundred.

b) How many prime numbers are there from one through one hundred?

c) What is the probability of randomly selecting a prime number from the numbers from one through one hundred?

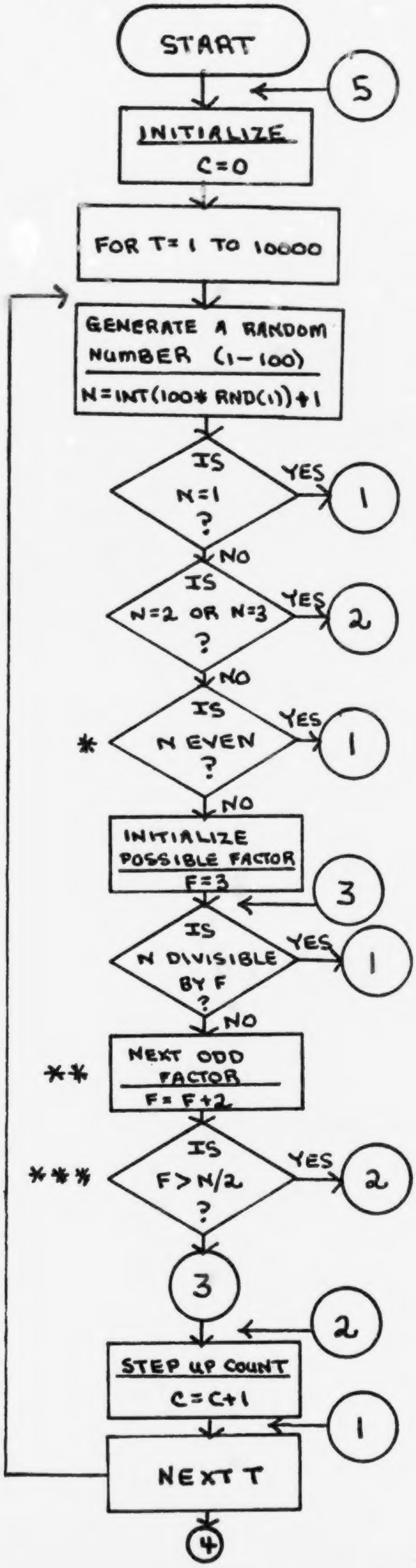
2. For the given problem, flowchart, and planned output, code a program to solve the problem.

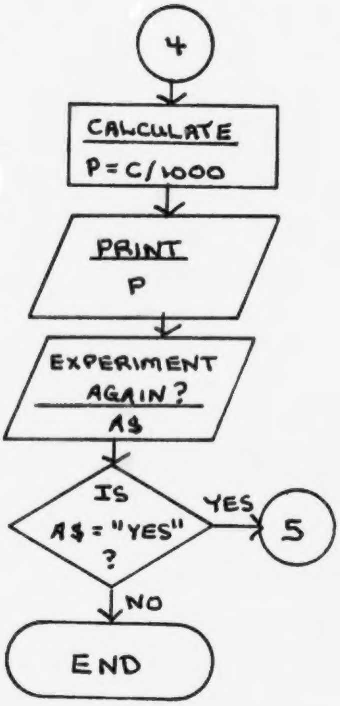
<u>Problem</u>: Have the computer simulate the random selection of ten thousand numbers from one through one hundred. Have it then determine and display the experimental probability of selecting a prime number.

<u>Planned Output</u>

```
SELECTING A PRIME NUMBER
FROM THE NUMBERS 1 THROUGH 100
(Skip 2 lines.)
THE PROBABILITY IS .2396
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? YES
(Skip 2 lines.)
THE PROBABILITY IS .2408
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? NO
(Skip 1 line.)
EXPERIMENTS CONCLUDED!
```

Flowchart


START
5
INITIALIZE
C=0
FOR T=1 TO 10000
GENERATE A RANDOM NUMBER (1–100)
N=INT(100*RND(1))+1
IS N=1 ?
YES
1
NO
IS N=2 OR N=3 ?
YES
2
NO
*
IS N EVEN ?
YES
1
NO
INITIALIZE POSSIBLE FACTOR
F=3
3
IS N DIVISIBLE BY F ?
YES
1
NO
**
NEXT ODD FACTOR
F=F+2
***
IS F>N/2 ?
YES
2
3
2
STEP UP COUNT
C=C+1
1
NEXT T
4


4
CALCULATE
P=C/1000
PRINT
P
EXPERIMENT AGAIN?
A$
IS A$="YES" ?
YES
5
NO
END

* Why are we not checking even numbers larger than 2 to see if they are prime?

** At this point, why are we only checking to see if odd numbers are possible factors?

*** Why are we only checking, as possible factors, odd numbers that are not larger than one-half of the number itself?

SUMMARY:

1. Did the experiments simulated on the computer in this lesson provide evidence that the probabilities assigned originally are correct?
2. What is the purpose of the random function in simulating a probability experiment?

HOMEWORK:

Complete the instructions for the following problem.

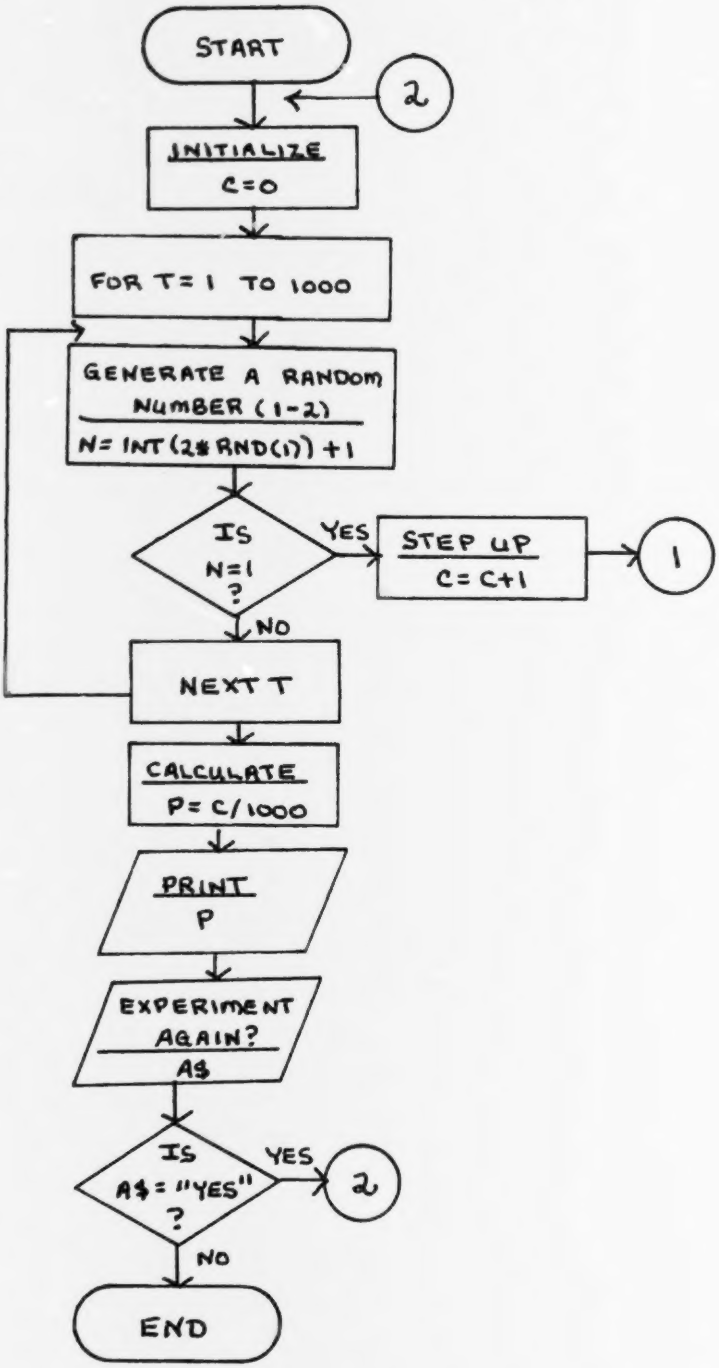
a) Write an analysis which includes the development of an algorithm.
b) Write a plan for a computer solution to the problem.
c) Develop a flowchart.
d) Design a planned output.
e) Code a program to solve the problem.

Problem: Have the computer simulate the tossing of a coin one thousand times. Have it then determine and display the experimental probability of getting a "head."

HOMEWORK SOLUTIONS:

(A suggested flowchart, planned output, and program follow. The analysis and plan can be gleaned from these.)

Flowchart


START
2
INITIALIZE
C=0
FOR T=1 TO 1000
GENERATE A RANDOM NUMBER (1-2)
N= INT (2* RND(1)) +1
IS N=1 ?
YES
STEP UP
C= C+1
1
NO
NEXT T
CALCULATE
P= C/1000
PRINT
P
EXPERIMENT AGAIN?
A$
IS A$ = "YES" ?
YES
2
NO
END

d) Planned Output

```
TOSSING A COIN
(Skip 2 lines.)
THE PROBABILITY OF A 'HEAD' IS .504
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? YES
(Skip 2 lines.)
THE PROBABILITY OF A 'HEAD' IS .499
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? NO
(Skip 1 line.)
EXPERIMENTS CONCLUDED!
```

e) Program

```
10  PRINT "TOSSING A COIN"
20  PRINT:PRINT
30  C=0
40  FOR T = 1 TO 1000
50  N = INT(2*RND(1))+1
60  IF N = 1 THEN C=C+1
70  NEXT T
80  P = C/1000
90  PRINT "THE PROBABILITY OF A 'HEAD' IS";P
100 PRINT
110 PRINT "DO YOU WISH TO DO THE EXPERIMENT AGAIN?"
120 INPUT A$
130 IF A$ = "YES" THEN 20
140 PRINT:PRINT "EXPERIMENTS CONCLUDED!"
150 END
```

Lesson: 57L

AIM: How do we simulate an experiment to determine the probability of obtaining a sum of ten on a throw of a pair of dice?

PERFORMANCE OBJECTIVES: The student will be able to...

1. list the outcomes in a probability experiment involving compound events;
2. plan (flowchart) the simulation of a probability experiment involving the throwing of a pair of dice;
3. design an output and code the program;
4. enter, run, and debug the program.

VOCABULARY: compound event, sum, random function (RND(1)), equally likely

MOTIVATION: List all the possible outcomes (sample space) when rolling two dice simultaneously.

DEVELOPMENT:

1a) Ask the students to list their responses to the motivation problem as follows.

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

1b) Say to the students that since two events (outcomes) occur at the same time, we call each of the listed outcomes a compound event.

1c) Ask, "What is the probability that the sum on the two dice will be ten?" Elicit that since there are thirty-six possible, equally likely outcomes and there are three ways ((4,6), (5,5), (6,4)) of obtaining a sum of ten, the probability is 3/10.

2. Say to the students that today they will plan, write, and execute the simulation described in the problem which follows.

Problem: Have the computer simulate the rolling of a pair of dice one thousand times. Have it then determine and display the experimental probability of the dice showing "numbers" whose sum is ten.

3. Elicit the analysis which follows from the students.

Analysis

Ask, "How can we get the computer to generate a random number from one through six in order to simulate the rolling of the first die? How can we get the computer to generate a random number from one through six in order to simulate the rolling of the second die?" Elicit D1 = INT(6*RND(1))+1 and D2 = INT(6*RND(1))+1.

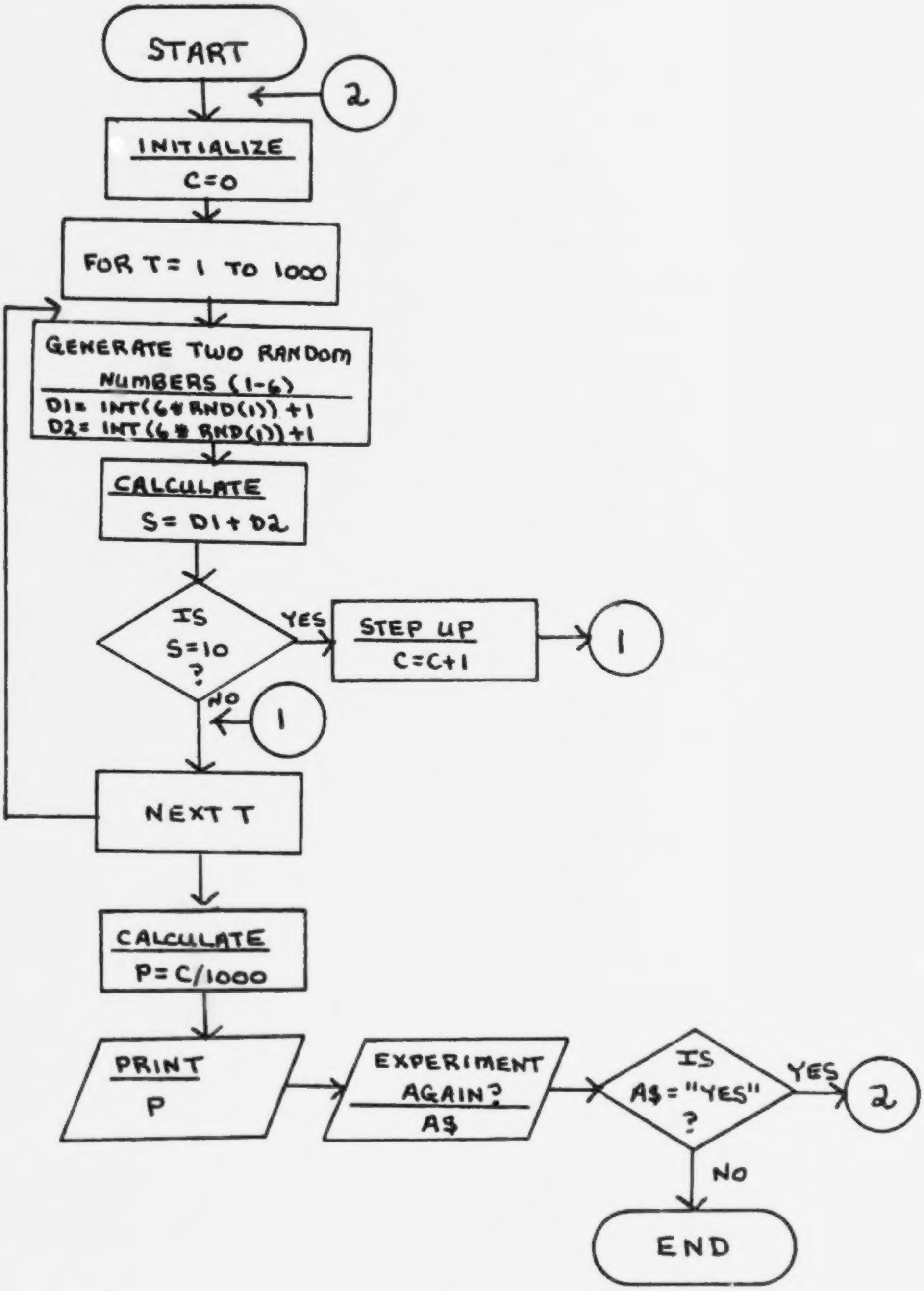
4. Ask the students to incorporate the algorithm into a plan as follows.

Plan

```
 1)  Initialize the counter for the sums of ten:  C=0.
 2)  Simulate the one thousand rolls:  FOR T = 1 TO 1000.
 3)  Simulate rolling a pair of dice:
     D1=INT(6*RND(1))+1
     D2=INT(6*RND(1))+1.
 4)  Calculate the sum:  S=D1+D2.
 5)  Check:  Is S=10?
     a) If yes, step up the count:  C=C+1. Then go to 6.
     b) If no, continue.
 6)  NEXT T.
 7)  Calculate the probability:  P=C/1000.
 8)  Print P.
 9)  Do the experiment again?  Answer yes or no:  A$.
10)  Check:  Is A$ = "YES"?
     a) If yes, go to 1.
     b) If no, continue.
11)  End.
```

5. Ask the students to prepare a flowchart solution as follows.

Flowchart



START
2
INITIALIZE
C=0
FOR T= 1 TO 1000
GENERATE TWO RANDOM NUMBERS (1-6)
D1= INT(6*RND(1))+1
D2= INT(6*RND(1))+1
CALCULATE
S= D1+D2
IS S=10 ?
YES
STEP UP
C=C+1
1
NO
1
NEXT T
CALCULATE
P=C/1000
PRINT
P
EXPERIMENT AGAIN?
A$
IS A$="YES" ?
YES
2
NO
END

6. Ask the students to design a planned output similar to the one which follows.

Planned Output

```
SUMMING DICE
(Skip 2 lines.)
THE PROBABILITY OF ROLLING A SUM OF TEN IS .297
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? YES
(Skip 2 lines.)
THE PROBABILITY OF ROLLING A SUM OF TEN IS .305
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? NO
(Skip 1 line.)
EXPERIMENTS CONCLUDED!
```

7. Ask the students to code the program which follows and run it several times.

Program

```
10  PRINT "SUMMING DICE"
20  PRINT:PRINT
30  C=0
40  FOR T = 1 TO 1000
50  D1=INT(6*RND(1))+1:D2=INT(6*RND(1))+1
60  S=D1+D2
70  IF S=10 THEN C=C+1
80  NEXT T
90  P=C/1000
100 PRINT "THE PROBABILITY OF ROLLING A SUM OF TEN IS";P
110 PRINT
120 PRINT "DO YOU WISH TO DO THE EXPERIMENT AGAIN?"
130 INPUT A$
140 IF A$ = "YES" THEN 20
150 PRINT:PRINT "EXPERIMENTS CONCLUDED!"
160 END
```

8. Ask the students who complete the program to work on the Application problem.

APPLICATION:

Modify the flowchart, planned output, and program written in today's lesson so that the computer will determine and display the experimental probability of the dice showing the same "numbers."

SUMMARY:

Did the experiment simulated on the computer in this lesson provide evidence that the probability assigned originally is correct?

HOMEWORK:

1. A quarter, a dime, and a penny are tossed simultaneously,
   a) List all the possible outcomes (sample space).
   b) What is the probability, expressed in decimal form, of tossing three "heads"?
2. Complete the instructions for the following problem.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan for a computer solution to the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program to solve the problem.

Problem: Have the computer simulate the tossing of three coins one thousand times. Have it determine and display the experimental probability of all three coins showing a "head."

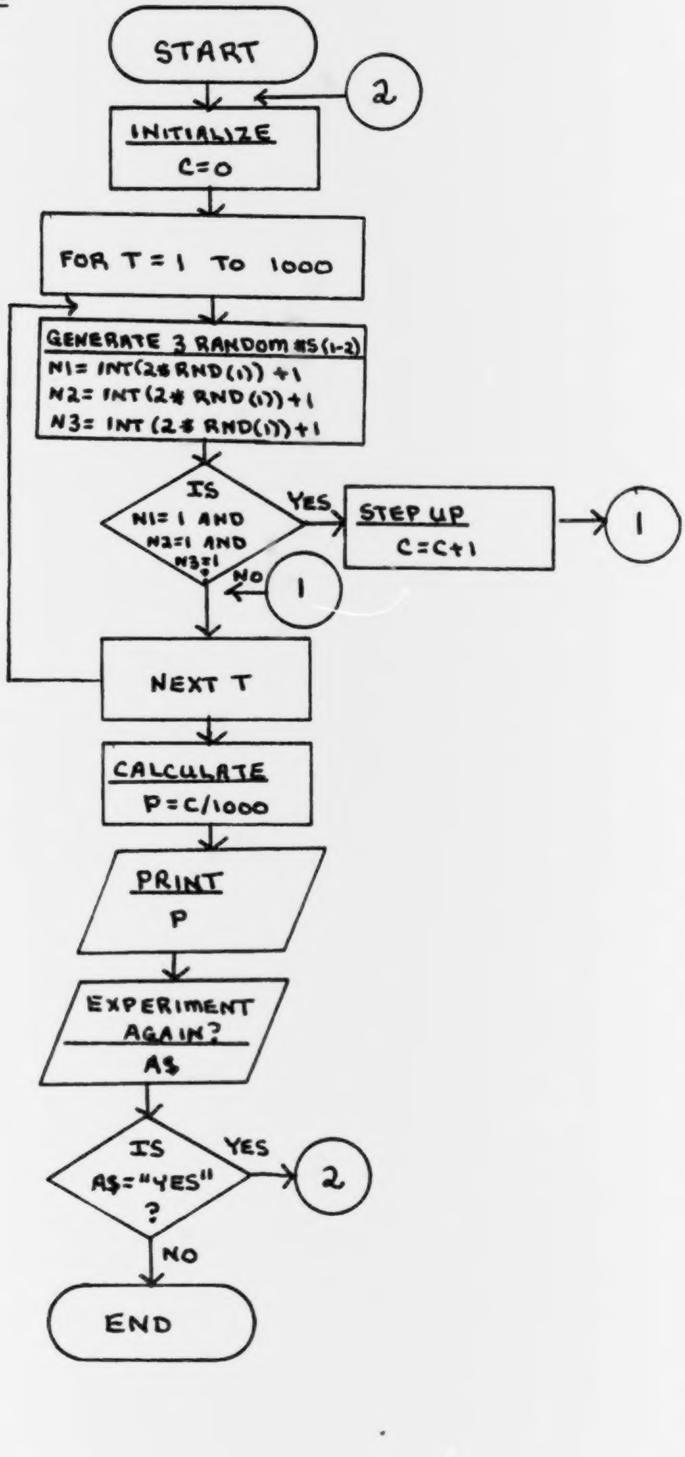
HOMEWORK SOLUTIONS:

1a) (H.H.H)
(H,H,T)
(H,T,H)
(H,T,T)
(T,H,T)
(T,T,H)
(T,H,H)
(T,T,T)

1b) .125

(A suggested flowchart, planned output, and program follow. The analysis and plan can be gleaned from these.)

c) Flowchart


START
2
INITIALIZE
C=0
FOR T=1 TO 1000
GENERATE 3 RANDOM #S (1-2)
N1= INT(2*RND(1)) +1
N2= INT(2*RND(1))+1
N3= INT(2*RND(1))+1
IS N1=1 AND N2=1 AND N3=1
YES
STEP UP
C=C+1
1
NO
1
NEXT T
CALCULATE
P=C/1000
PRINT
P
EXPERIMENT AGAIN?
A$
IS A$="YES" ?
YES
2
NO
END

d) Planned Output

```
TOSSING THREE COINS
(Skip 2 lines.)
THE PROBABILITY OF TOSSING ALL 'HEADS' IS .121
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? YES
(Skip 2 lines.)
THE PROBABILITY OF TOSSING ALL 'HEADS' IS .129
(Skip 1 line.)
DO YOU WISH TO DO THE EXPERIMENT AGAIN?
? NO
(Skip 1 line.)
EXPERIMENTS CONCLUDED!
```

e) Program

```
10  PRINT "TOSSING THREE COINS"
20  PRINT:PRINT
30  C=0
40  FOR T=1 TO 1000
50  N1 = INT(2*RND(1))+1
60  N2 = INT(2*RND(1))+1
70  N3 = INT(2*RND(1))+1
80  IF N1=1 AND N2=1 AND N3=1 THEN C=C+1
90  NEXT T
100 P=C/1000
110 PRINT "THE PROBABILITY OF TOSSING ALL 'HEADS' IS"; P
120 PRINT
130 PRINT "DO YOU WISH TO DO THE EXPERIMENT AGAIN?"
140 INPUT A$
150 IF A$ = "YES" THEN 20
160 PRINT:PRINT "EXPERIMENTS CONCLUDED!"
170 END
```

Lesson: 58

AIM: How can a computer sort a list of numbers?

PERFORMANCE OBJECTIVES: The student will be able to...

1. arrange rational numbers in ascending order;
2. simulate a procedure used by the computer to sort numbers (the bubble sort).

VOCABULARY: bubble sort, pass, rational number, interchange, ascending order

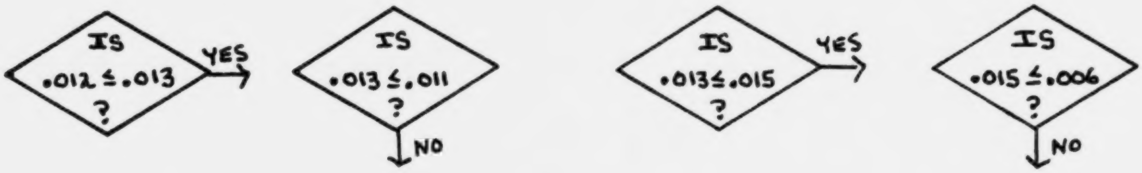
MOTIVATION: Arrange each of the following sets of rational numbers in ascending order.

1) .012, .013, .011, .015, .006
2) .2, .18, .09, .11, .1
3) .101, .011, .111, .11, .1

DEVELOPMENT:

1. As you elicit responses to the motivation examples, review the procedure for ordering rational numbers. Ask the students to recall that they wrote programs to arrange three rational numbers in ascending order earlier in the term (Lesson 2L, Part II). Today they will be examining a more efficient technique for ordering numbers, one which rearranges the original list while making the comparisons. For example, to order the numbers from smallest to largest (ascending order), the computer will begin by comparing the first two numbers. If the first is not the smaller, it will interchange the numbers. If the first is the smaller, the computer will compare the second and third numbers in the list, and so on. Let us examine more closely how this can be accomplished using the five numbers listed in example 1 (motivation).

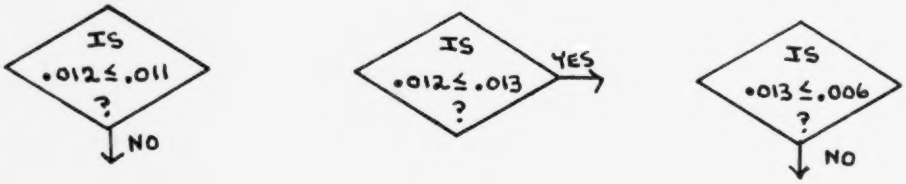
2. We begin our comparisons using the first two numbers.



Original List	List Unchanged	Interchange .013 and .011	List Unchanged	Interchange .015 and .006
.012	**.012**	.012	.012	.012
.013	**.013**	**.011**	.011	.011
.011	**.011**	**.013**	.013	.013
.015	.015	**.015**	**.015**	**.006**
.006	.006	.006	**.006**	**.015**

We have just completed the first pass. Ask, "In terms of arranging the numbers in ascending order, what have we accomplished in the first pass?" Elicit that the largest number has "fallen" to the bottom as the smaller numbers are beginning to "float" to the top. Ask, "How many comparisons were made in the first pass?" Ask the students to note that each of the first four numbers was compared to the next number.

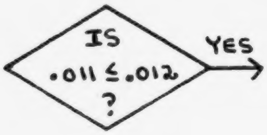
3. Let us now make the second pass. This means that we begin again with the first two numbers. Ask, "Which number don't we need to involve in this pass because it is already where we want it?" Elicit that in this pass, we will stop after we reach the number that is second from the bottom.

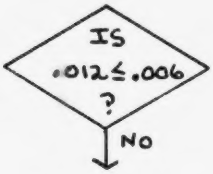


After The First Pass	Interchange .012 and .011	List Unchanged	Interchange .013 and .006
.012	**.011**	.011	.011
.011	**.012**	**.012**	.012
.013	**.013**	**.013**	**.006**
.006	.006	**.006**	**.013**
.015	.015	.015	.015

Ask, "In terms of ordering, what have we accomplished in the second pass?" Elicit that the two largest numbers are at the bottom in the correct order while the smaller numbers are continuing to "float" to the top. Ask, "How many comparisons were made in the second pass?" Ask the students to note that each of the first three numbers was compared to the next number.

4. We begin the third pass, again with the first two numbers. Ask, "Which numbers don't we need to involve in this pass because they are already where we want them?" Elicit that in this pass, we will stop after we reach the number that is third from the bottom.





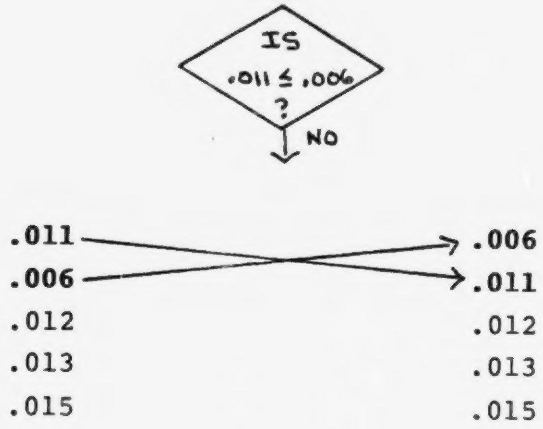
After The Second Pass	List Unchanged	Interchange .012 and .006
.011	**.011**	.011
.012	**.012**	**.006**
.006	**.006**	**.012**
.013	.013	.013
.015	.015	.015

Ask, "In terms of ordering, what have we accomplished in the third pass?" Elicit that the three largest numbers are at the bottom in the correct order while the smallest two numbers have "floated" to the top. Ask, "How many comparisons were made in the third pass?" Ask the students to note that each of the first two numbers was compared to the next number.

5. We begin our fourth pass, again with the first two numbers. Ask, "Which numbers don't we need to involve in this pass because they are already where we want them?" Elicit that in this pass, we will stop after comparing the first and second (fourth from the bottom) numbers.

After The Third Pass

Interchange .011 and .006



The numbers are now in ascending order.
Ask, "How many comparisons were made in the fourth pass?" Ask the students to note that only the first number was compared to the second (the next number). In two days, we will translate this scheme into BASIC code. Tomorrow we will discuss a new type of variable which will permit us to compare a number to the next number in a list more efficiently than the numeric variables we have been using.

6. Ask the students to complete the Application example.

APPLICATION:

Show how the procedure outlined in today's lesson would arrange each set of five numbers in examples 2 and 3 (motivation). Be sure to show the list for each comparison made.

SUMMARY:

Why do you think the procedure outlined to today's lesson for arranging numbers in numerical order is called the "bubble sort"?

HOMEWORK:

1. Arrange each of the following sets of rational numbers in ascending order.

   a) .04, .014, .01, .004, .104
   b) .7, .77, .077, .707, .007
   c) .0025, .005, .0505, .0205, .025
   d) 2.6, 2.606, 2.66, 2.066, 2.006
   e) 1.9, 2.105, 2.15, 2.05, 2

2. Show how the bubble sort would arrange each of the sets of five numbers in example 1. Be sure to show the list for each comparison made.

HOMEWORK SOLUTIONS:

1. a) .004, .01, .014, .04, .104
   b) .007, .077, .7, .707, .77
   c) .0025, .005, .0205, .025, .0505
   d) 2.006, 2.066, 2.6, 2.606, 2.66
   e) 1.9, 2, 2.05, 2.105, 2.15

2. See the development in this lesson for a model for the responses for this question.

Lesson: 59

AIM: What is a subscripted variable?

PERFORMANCE OBJECTIVES: The student will be able to...

1. define a subscripted variable (array);
2. trace programs in which subscripted variables are used.

VOCABULARY: subscripted variable, single-dimensional array, subscript, dimension statement
Symbols: A(X), DIM A(100)

MOTIVATION: Show the complete output for the program below.

Program

```
10 READ A, B, C, D, E, F, G, H
20 PRINT A; B; C; D; E; F; G; H
30 DATA 5, 10, 15, 20, 25, 30, 35, 40
40 END
```

DEVELOPMENT:

1. After eliciting a response to the motivation problem, say to the students that the following program can be used to produce the same output.

Program

```
10 READ A(0), A(1), A(2), A(3), A(4), A(5), A(6), A(7)
20 PRINT A(0); A(1); A(2); A(3); A(4); A(5); A(6); A(7)
30 DATA 5, 10, 15, 20, 25, 30, 35, 40
40 END
```

The second program uses a subscripted variable or single-dimensional array. A subscripted variable or array is an ordered storage place for data. Each number in the data is assigned an order; e.g.,

A(0)	A(1)	A(2)	A(3)	A(4)	A(5)	A(6)	A(7)
5	10	15	20	25	30	35	40

Say to the students that this will make it possible for us to have the computer easily compare a number to the next number when we write a program using the bubble sort tomorrow.

The subscripted variable is read A sub 0, A sub 1, A sub 2, ... where 0, 1, 2, ... are called the values of the <u>subscript</u>. The subscript of the variable always appears in parentheses after the variable.

2a) Say to the students that since the values of the subscript are whole numbers ranging from zero through seven, they can be assigned more efficiently using a FOR/NEXT loop as follows.

<u>Program</u>

```
10 FOR X = 0 TO 7
20 READ A(X)
30 PRINT A(X);
40 NEXT X
50 DATA 5, 10, 15, 20, 25, 30, 35, 40
60 END
```

2b) Ask the students to show the output for the program as follows. Stress what occurs in the memory.

<u>Screen</u>

```
RUN
5  10  15  20  25  30  35  40
```

<u>Memory</u>

X	A(0)	A(1)	A(2)	A(3)	A(4)	A(5)	A(6)	A(7)
0	5	10	15	20	25	30	35	40
~~1~~								
~~2~~								
~~3~~								
~~4~~								
~~5~~								
~~6~~								
7								
8								

3a) Say to the students that when the computer encounters a subscripted variable, A(X), it automatically reserves room for eleven pieces of data in A(0), A(1), A(2), ..., A(10). If the programmer wants to make the value of the subscript greater than ten, providing room for additional data, he/she must give the computer that information at the beginning of the program in a DIMension statement.

3b) Ask the students to show the output for the program which follows; it uses the DIMension statement. Again, stress what occurs in the memory.

Program

```
10 DIM B(15)
20 FOR X = 1 TO 12
30 READ B(X)
40 NEXT X
50 FOR Y = 12 TO 1 STEP -1
60 PRINT B(Y);
70 NEXT Y
80 DATA 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5
90 END
```

Screen

```
RUN
5  10  25  20  25  30  35  40  45  50  55  60
```

Memory

X	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)	B(7)	B(8)	B(9)	B(10)	B(11)	B(12)	Y
~~1~~	60	55	50	45	40	35	30	25	20	15	10	5	~~12~~
~~2~~													~~11~~
~~3~~													~~10~~
~~4~~													~~9~~
~~5~~													~~8~~
~~6~~													~~7~~
~~7~~													~~6~~
~~8~~													~~5~~
~~9~~													~~4~~
~~10~~													~~3~~
~~11~~													~~2~~
~~12~~													~~1~~
13													0

3c) Say to the students that DIM B(15) "tells" the computer to reserve room for sixteen pieces of data stored in B(0), B(1), B(2), ..., B(15). Ask, "How many pieces of data are used in the program?" Ask the students to note that the programmer can reserve more memory space than the computer uses in executing the program; however, he/she cannot reserve less memory space.

4. Ask the students to complete the Applications problems. Ask them to identify the subscripted variable(s), subscript(s) and the range of values for the subscript(s) in each.

APPLICATIONS:

Show the complete output for each program.

1.

```
10 FOR X = 1 TO 4
20 READ C(X)
30 PRINT C(X);
40 NEXT X
50 DATA 2,4,6,8
60 END
```

2.

```
10 FOR Y = 1 TO 3
20 READ D(Y)
30 NEXT Y
40 FOR Z = 1 TO 3
50 PRINT Z, D(Z)
60 NEXT Z
70 DATA 3,5,7
80 END
```

3.

```
10 FOR R = 1 TO 2
20 READ E(R), F(R)
30 NEXT R
40 DATA 8,7,6,5
50 FOR R = 1 TO 2
60 PRINT E(R), F(R)
70 NEXT R
80 END
```

4.

```
10   FOR P = 1 TO 4
20   READ G(P), H(P)
30   NEXT P
40   DATA 1,4,3,9,6,16,10,25
50   FOR P = 1 TO 4
60   J(P) = G(P) + H(P)
70   NEXT P
80   FOR P = 1 TO 4
90   PRINT G(P), H(P), J(P)
100  NEXT P
110  END
```

5.

```
10 DIM K(12)
20 FOR L = 1 TO 12
30 K(L) = 12-L
40 NEXT L
50 FOR L=2 TO 12 STEP 2
60 PRINT K(L);
70 NEXT L
80 END
```

6.

```
10   FOR S = 1 TO 4
20   READ N(S)
30   NEXT S
40   FOR S = 1 TO 3
50   IF N(S)<=N(S+1) THEN 80
60   PRINT N(S)
70   GOTO 90
80   PRINT N(S+1)
90   NEXT S
100  DATA 9,36,35,7
110  END
```

SUMMARY:

1. What is a subscripted variable?
2. What is the function of a DIMension statement?

HOMEWORK:

Show the output for each program.

1.
```
10 FOR A = 1 TO 7
20 READ Z(A)
30 NEXT A
40 FOR A = 1 TO 7 STEP 2
50 PRINT Z(A)
60 NEXT A
70 DATA 1,3,6,10,15,21,28
80 END
```

2.
```
10 FOR B = 1 TO 5
20 Y(B) = 3*B
30 NEXT B
40 FOR C = 1 TO 5
50 PRINT C, Y(C)
60 NEXT C
70 END
```

3.
```
10 FOR E = 1 TO 4
20 R(E)= 1 + 2*E
30 NEXT E
40 FOR E = 4 TO 1 STEP-1
50 PRINT R(E);
60 NEXT E
70 END
```

4.
```
10 FOR G = 1 TO 7
20 P(G) = 7-G
30 NEXT G
40 FOR G = 1 TO 7
50 PRINT P(G)
60 NEXT G
70 END
```

5.
```
10   FOR D = 1 TO 3
20   READ W(D), T(D)
30   NEXT D
40   DATA 4,2,9,6,16,12
50   FOR D = 1 TO 3
60   S(D)= W(D)-T(D)
70   NEXT D
80   FOR D = 1 TO 3
90   PRINT W(D), T(D), S(D)
100  NEXT D
110  END
```

6.
```
10   C=0
20   FOR F = 1 TO 8
30   READ Q(F)
40   NEXT F
50   FOR F = 1 TO 8
60   IF Q(F) = 7 THEN C=C+1
70   NEXT F
80   DATA 7,3,7,7,5,7,7,7
90   PRINT "COUNT: ";C
100  END
```

HOMEWORK SOLUTIONS:

1. Screen

```
RUN
1
6
15
28
```

Memory

A	Z(1)	Z(2)	Z(3)	Z(4)	Z(5)	Z(6)	Z(7)
1	1	3	6	10	15	21	28
~~2~~							
~~3~~							
~~4~~							
~~5~~							
~~6~~							
~~7~~							
~~8~~							
~~1~~							
~~3~~							
~~5~~							
~~7~~							
9							

2. Screen

```
RUN
1       3
2       6
3       9
4       12
5       15
```

Memory

B	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	C
~~1~~	3*1=	3*2=	3*3=	3*4=	3*5=	~~1~~
~~2~~	3	6	9	12	15	~~2~~
~~3~~						~~3~~
~~4~~						~~4~~
~~5~~						~~5~~
6						6

3. Screen

```
RUN
9 7 5 3
```

Memory

E	R(1)	R(2)	R(3)	R(4)
~~1~~	1+2*1=	1+2*2=	1+2*3=	1+2*4=
~~2~~	1+2=	1+4=	1+6=	1+8=
~~3~~	3	5	7	9
~~4~~				
~~5~~				
~~4~~				
~~3~~				
~~2~~				
~~1~~				
0				

4. Screen

```
RUN
6
5
4
3
2
1
0
```

Memory

G	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)
~~1~~	7-1=	7-2=	7-3=	7-4=	7-5=	7-6=	7-7=
~~2~~	6	5	4	3	2	1	0
~~3~~							
~~4~~							
~~5~~							
~~6~~							
~~7~~							
~~8~~							
~~1~~							
~~2~~							
~~3~~							
~~4~~							
~~5~~							
~~6~~							
~~7~~							
8							

5. Screen

```
RUN
4       2       2
9       6       3
16      12      4
```

Memory

D	W(1)	T(1)	W (2)	T(2)	W(3)	T(3)	S(1)	S(2)	S(3)
~~1~~	4	2	9	6	16	12	4-2= 2	9-6= 3	16-12= 4
~~2~~									
~~3~~									
~~4~~									
~~1~~									
~~2~~									
~~3~~									
~~4~~									
~~1~~									
~~2~~									
~~3~~									
4									

6. Screen

```
RUN
COUNT: 6
```

Memory

C	F	Q(1)	Q(2)	Q(3)	Q(4)	Q(5)	Q(6)	Q(7)	Q(8)
~~0~~	~~1~~	7	3	7	7	5	7	7	7
~~1~~	~~2~~								
~~2~~	~~3~~								
~~3~~	~~4~~								
~~4~~	~~5~~								
~~5~~	~~6~~								
6	~~7~~								
	~~8~~								
	~~9~~								
	~~1~~								
	~~2~~								
	~~3~~								
	~~4~~								
	~~5~~								
	~~6~~								
	~~7~~								
	~~8~~								
	9								

Lesson: 60L

AIM: How do we write a program that will sort a list of numbers?

PERFORMANCE OBJECTIVES: The student will be able to...

1. incorporate the bubble sort and the subscripted variable into a plan (flowchart) for having the computer arrange rational numbers in ascending order;
2. design a planned output;
3. code a program;
4. enter, run, and debug the program.

VOCABULARY: ascending order, descending order, bubble sort, subscripted variable, pass

MOTIVATION: Develop a computer solution for the following problem.

> Problem: Use the bubble sort to have the computer sort five numbers and display them in ascending order.

DEVELOPMENT:

1a) Develop the plan which follows with the students.

Plan

1) Enter five numbers, storing them in N(1), N(2), N(3), N(4), and N(5).
2) Use N(L) to represent a particular number in the list.
3) Use N(L+1) to represent the next number in the list.
4) Initialize a counter to keep track of the four passes: P=1.
5) Set up a FOR/NEXT loop to move down the list making the comparisons between a number and the next number:

FOR L=1 TO $\underbrace{5-P}$

For each pass, we do not use the memory position that will contain the latest number which is arranged in the proper order.

```
6) Check:  Is N(L) ≤ N(L+1)?
   a)  If yes, then go to 8.
   b)  If no, continue.
7) Interchange the contents of N(L) and N(L+1).
8) NEXT L.
9) Step up to the next pass: P=P+1.
10) Check:  Is P>4?
   a)  If yes, then go to 12.
   b)  If no, continue.
11) Go to 5.
12) Display the five numbers in ascending order.
13) End.
```

1b) Ask the following questions as you develop the plan.

i) Into what type of variable will we enter our original list of five numbers?

ii) If memory cell L stores the number for a particular position in the list, how do we represent the number for the next position in the list?

iii) How many passes do we want the computer to make?

iv) For each pass, how many comparisons do we want the computer to make?

v) Ask the students to develop and then examine the table which follows.

Pass	Number of Comparisons Made
1	4
2	3
3	2
4	1

If there are five numbers stored and memory cell P stores the number for each pass, how do we represent the number of comparisons made during that pass?

vi) What do we have the computer check about a particular number in the list and the next number?

vii) If a particular number in the list *is not* less than or equal to the next number in the list, what do we want the computer to do before it goes on to the next number in the list?

viii) If a particular number in the list *is* less than or equal to the next number in the list, what do we want the computer to do?

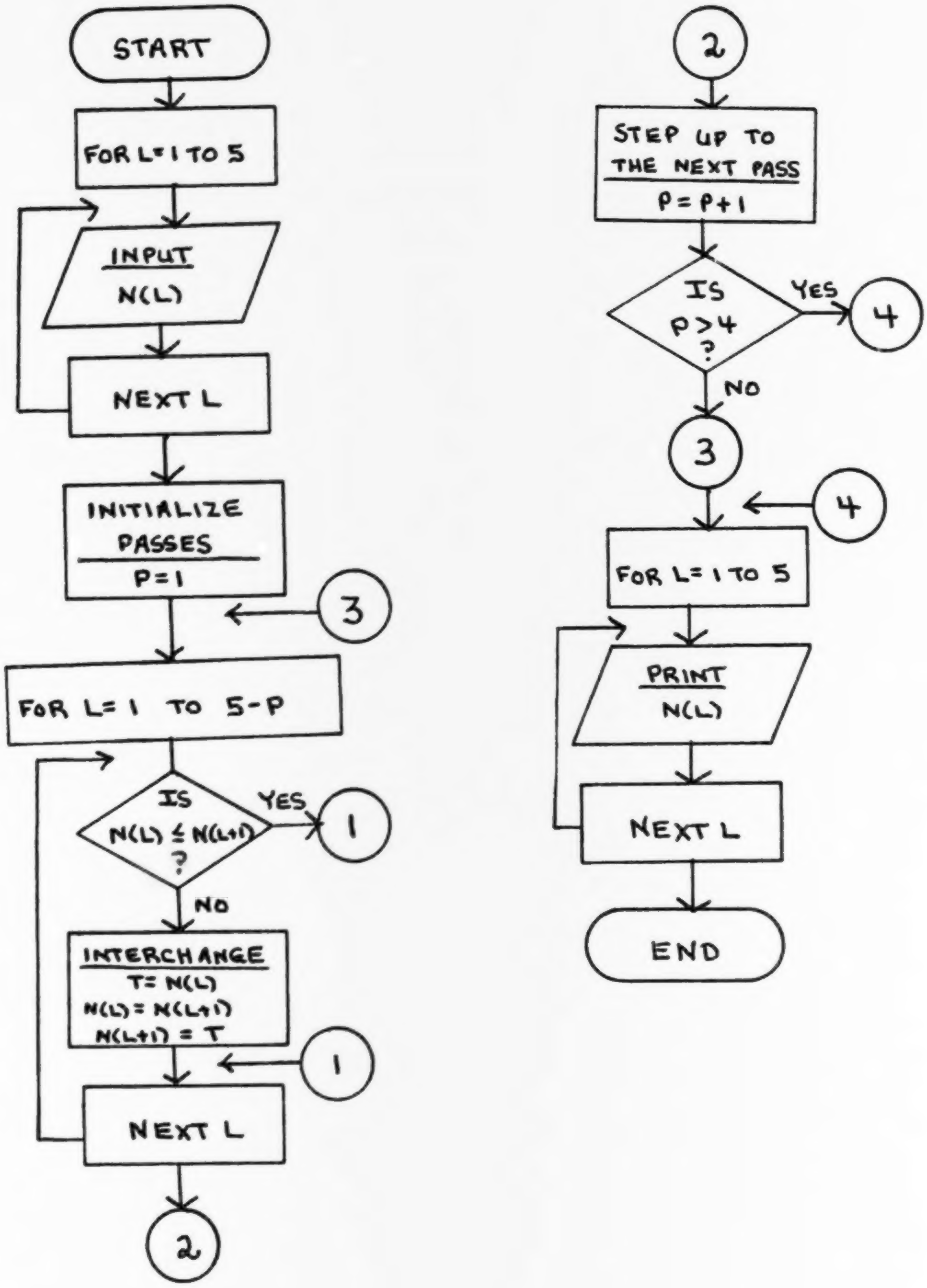
ix) Once a pass is complete, how does the computer get to the next pass number?

x) When should the computer stop the process?

xi) Once the numbers are arranged in ascending order, what do we want the computer to do?

2a) Ask the students to develop the flowchart which follows.

Flowchart


START
FOR L=1 TO 5
INPUT
N(L)
NEXT L
INITIALIZE
PASSES
P=1
3
FOR L=1 TO 5-P
IS
N(L) ≤ N(L+1)
?
YES
1
NO
INTERCHANGE
T= N(L)
N(L)= N(L+1)
N(L+1) = T
1
NEXT L
2
2
STEP UP TO
THE NEXT PASS
P=P+1
IS
P>4
?
YES
4
NO
3
4
FOR L= 1 TO 5
PRINT
N(L)
NEXT L
END

2b) Ask the students to complete Applications problem 1.

3. Ask the students to design an output as follows.

Planned Output

```
ARRANGING NUMBERS IN ASCENDING ORDER
(Skip 2 lines.)
ENTER FIVE NUMBERS.
? 5
? 4.6
? 3.9
? 4.1
? 4
(Skip 1 line.)
THE LIST IN ASCENDING ORDER IS...
3.9
4
4.1
4.6
5
```

4. Ask the students to code and run the program; ask them to correct any errors they make. Circulate around the laboratory assisting those students who need help and offering support to the others.

Program

```
10  PRINT "ARRANGING NUMBERS IN ASCENDING ORDER"
20  PRINT:PRINT
30  PRINT "ENTER FIVE NUMBERS."
40  FOR L=1 TO 5
50  INPUT N(L)
60  NEXT L
70  P=1
80  FOR L=1 TO 5-P
90  IF N(L)<=N(L+1) THEN 110
100 T=N(L): N(L)=N(L+1): N(L+1)=T
110 NEXT L
120 P=P+1
130 IF P>4 THEN 150
140 GOTO 80
150 PRINT:PRINT "THE LIST IN ASCENDING ORDER IS..."
160 FOR L= 1 TO 5
170 PRINT N(L)
180 NEXT L
190 END
```

5. Ask the students who complete the program to work on Applications examples 2 and 3.

APPLICATIONS:

1. Trace the flowchart developed in this lesson using the following list of five numbers: 5, 4.6, 3.9, 4.1, 4.
2. Modify the program written in today's lesson so that it arranges and then displays fifteen numbers in ascending order.
   (Note to the teacher: Be sure that the students add the statement DIM N(15) to the beginning of the program.)
3. Modify the program written in today's lesson so that it arranges and then displays five numbers in <u>descending</u> order.

SUMMARY:

1. How does the use of the subscripted variable allow the computer to sort numbers more efficiently?
2. Why is the program written in today's lesson more efficient in sorting numbers than the one developed at the beginning of the term?
3. If no interchanges are made during a particular pass, what does that mean? Does the computer "know" that with our program?

HOMEWORK:

(Allow the students at least one week to solve one of the problems listed below.)

Complete the instructions for the following problems.

a) Write an analysis which includes the development of an algorithm.
b) Write a plan for a computer solution to the problem.
c) Develop a flowchart.
d) Design a planned output.
e) Code a program to solve the problem.

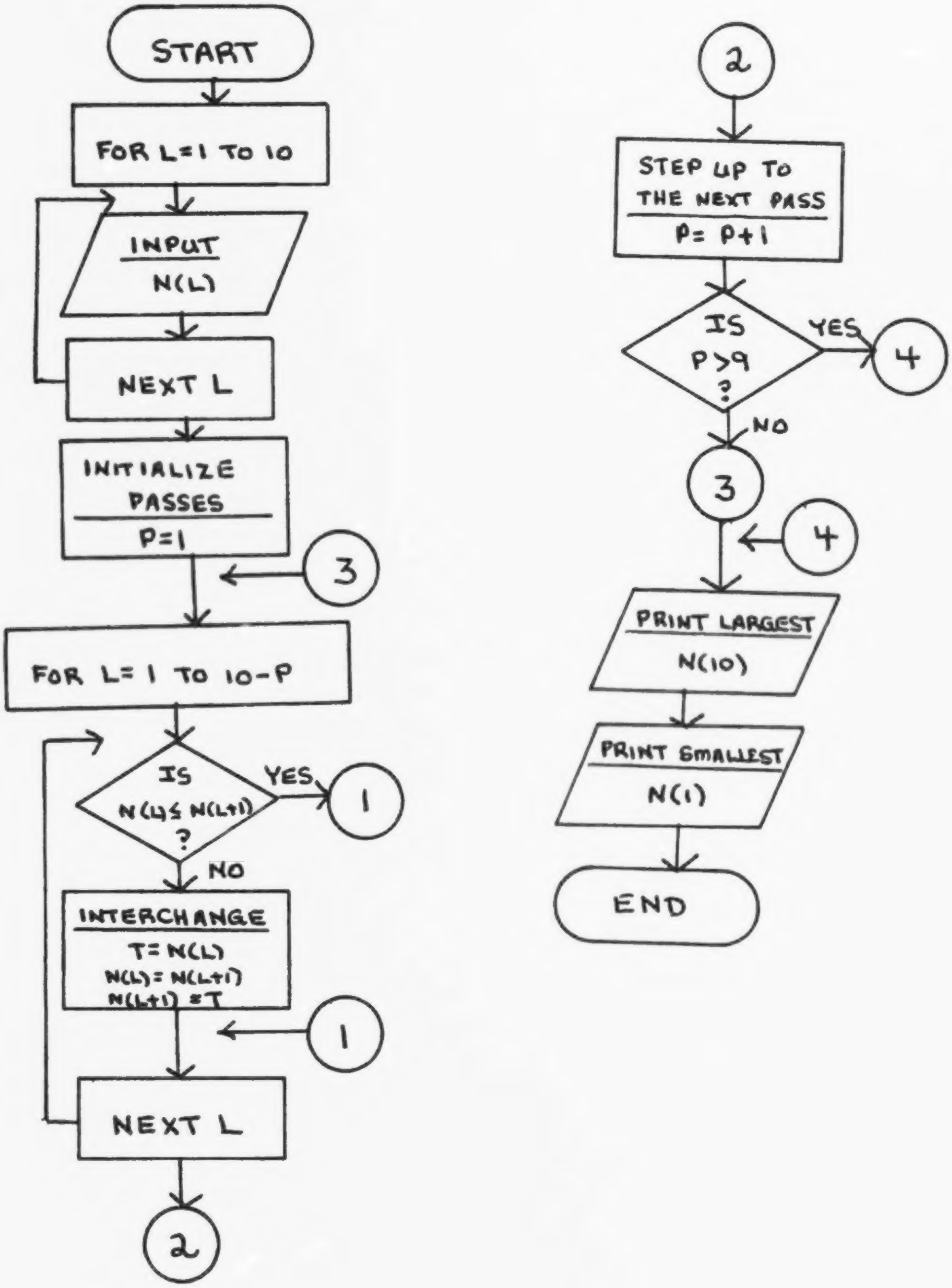
Problems:

1. Enter ten numbers, Have the computer display the largest and then the smallest number.
2. At a gymnastics meet, there are seven judges. Each judge records a score from one through ten. Each contestant's final score is determined by eliminating the highest and lowest scores and then finding the arithmetic mean of the remaining scores. Have the computer determine and display a contestant's final score.
3. Have the computer arrange **A** numbers in ascending order where **A**, the amount of numbers, is the user's choice. (Be sure to provide enough memory space for amounts larger than ten.)

HOMEWORK SOLUTIONS:

(Suggested flowcharts, planned outputs, and programs are shown. The analyses and plans can be gleaned from these.)

1c) Flowchart


START
FOR L=1 TO 10
INPUT
N(L)
NEXT L
INITIALIZE PASSES
P=1
3
FOR L= 1 TO 10-P
IS N(L)≤N(L+1) ?
YES
1
NO
INTERCHANGE
T=N(L)
N(L)=N(L+1)
N(L+1)=T
1
NEXT L
2
2
STEP UP TO THE NEXT PASS
P=P+1
IS P>9 ?
YES
4
NO
3
4
PRINT LARGEST
N(10)
PRINT SMALLEST
N(1)
END

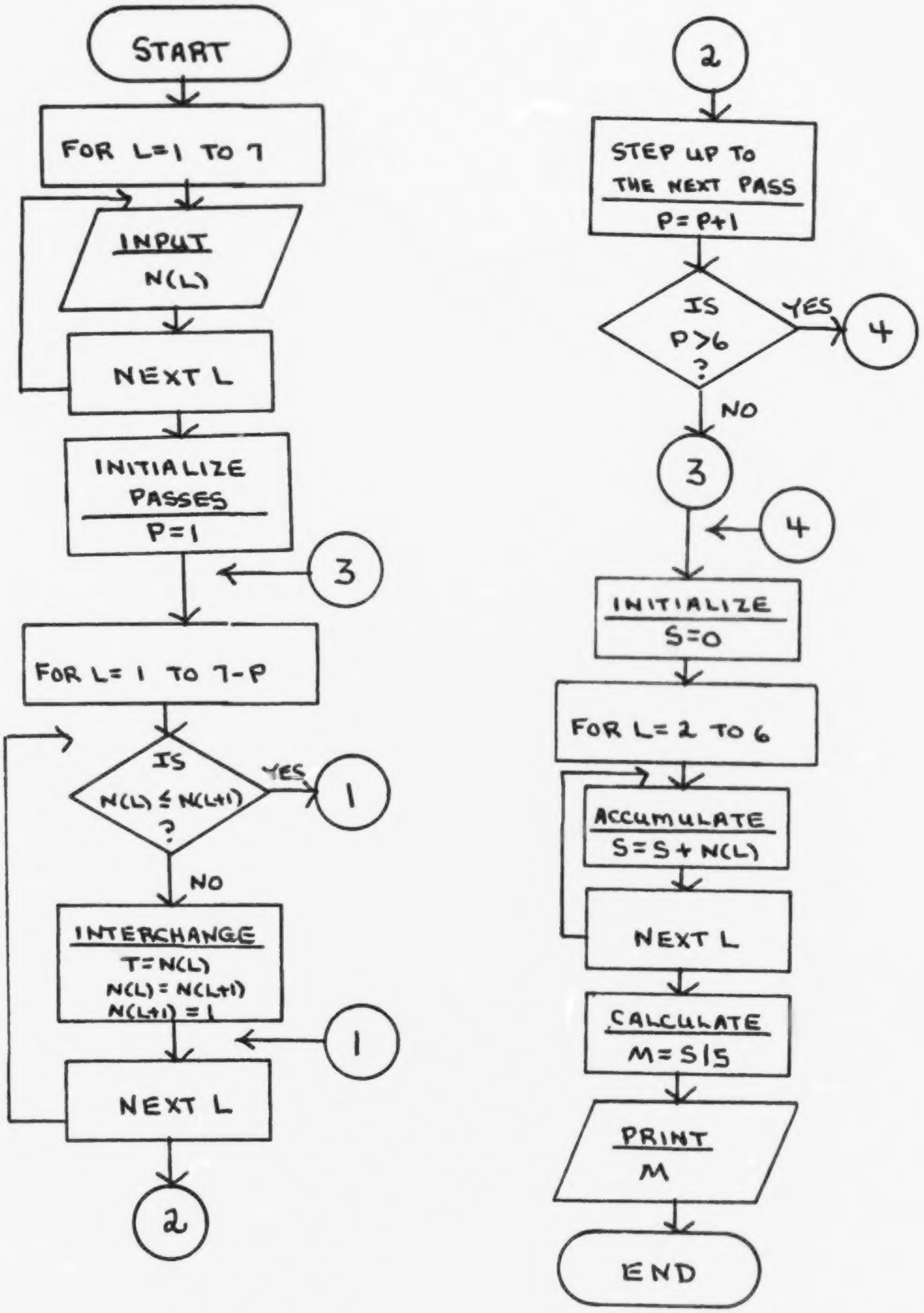
1d) Planned Output

```
FINDING THE LARGEST AND SMALLEST
(Skip 1 line.)
ENTER TEN NUMBERS.
? 3
? 2.1
? 2.7
? 2
? 2.5
? 2.2
? 2.9
? 2.3
? 2.6
? 2.8
THE LARGEST NUMBER IS 3
THE SMALLEST NUMBER IS 2
```

1e) Program

```
10  PRINT "FINDING THE LARGEST AND SMALLEST":PRINT
20  PRINT "ENTER TEN NUMBERS."
30  FOR L = 1 TO 10
40  INPUT N(L)
50  NEXT L
60  P=1
70  FOR L=1 TO 10-P
80  IF N(L) < = N(L+1) THEN 100
90  T=N(L): N(L)=N(L+1): N(L+1)=T
100 NEXT L
110 P=P+1
120 IF P>9 THEN 140
130 GOTO 70
140 PRINT "THE LARGEST NUMBER IS"; N(10)
150 PRINT "THE SMALLEST NUMBER IS"; N(1)
160 END
```

2c) Flowchart


START
FOR L=1 TO 7
INPUT
N(L)
NEXT L
INITIALIZE
PASSES
P=1
3
FOR L= 1 TO 7-P
IS
N(L) ≤ N(L+1)
?
YES
1
NO
INTERCHANGE
T=N(L)
N(L) = N(L+1)
N(L+1) = 1
1
NEXT L
2
2
STEP UP TO
THE NEXT PASS
P=P+1
IS
P>6
?
YES
4
NO
3
4
INITIALIZE
S=0
FOR L= 2 TO 6
ACCUMULATE
S=S+N(L)
NEXT L
CALCULATE
M=S/5
PRINT
M
END

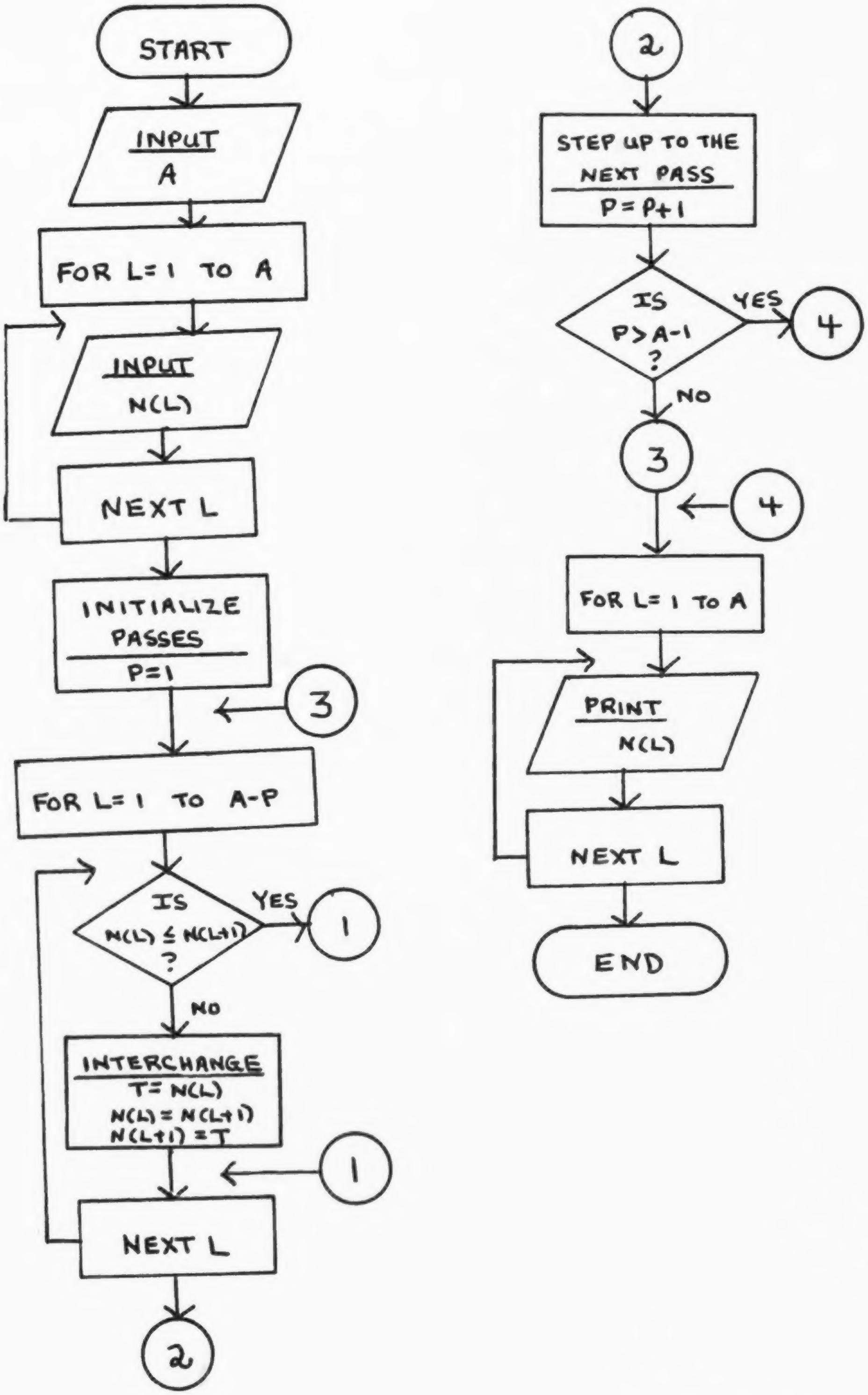
2d) Planned Output

```
GYMNASTICS MEET SCORE
(Skip 1 line.)
ENTER THE SCORES (1-10) FOR THE SEVEN JUDGES.
? 9.8
? 10
? 9.5
? 9.9
? 9
? 9.7
? 9.6
(Skip 1 line.)
THE CONTESTANT'S FINAL SCORE IS 9.7
```

2e) Program

```
10  PRINT "GYMNASTICS MEET SCORE":PRINT
20  PRINT "ENTER THE SCORES (1-10) FOR THE SEVEN JUDGES."
30  FOR L = 1 TO 7
40  INPUT N(L)
50  NEXT L
60  P=1
70  FOR L = 1 TO 7-P
80  IF N(L) < = N(L+1) THEN 100
90  T=N(L): N(L)=N(L+1): N(L+1)=T
100 NEXT L
110 P=P+1
120 IF P > 6 THEN 140
130 GOTO 70
140 S=0
150 FOR L=2 TO 6
160 S=S+N(L)
170 NEXT L
180 M=S/5
190 PRINT:PRINT "THE CONTESTANT'S FINAL SCORE IS";M
200 END
```

3c) Flowchart


START
INPUT
A
FOR L=1 TO A
INPUT
N(L)
NEXT L
INITIALIZE
PASSES
P=1
3
FOR L=1 TO A-P
IS
N(L) ≤ N(L+1)
?
YES
1
NO
INTERCHANGE
T= N(L)
N(L) = N(L+1)
N(L+1) =T
1
NEXT L
2
2
STEP UP TO THE
NEXT PASS
P=P+1
IS
P>A-1
?
YES
4
NO
3
4
FOR L=1 TO A
PRINT
N(L)
NEXT L
END

3d) Planned Output

```
ARRANGING NUMBERS IN ASCENDING ORDER
(Skip 1 line.)
HOW MANY NUMBERS DO YOU WANT TO SORT? 12
(Skip 1 line.)
ENTER THE 12 NUMBERS.
? 1.8
? 3
? 2.1
? 2.7
? 2
? 2.5
? 2.2
? 2.9
? 2.3
? 2.6
? 2.8
? 3.2
(Skip 1 line.)
THE LIST IN ASCENDING ORDER IS ...
1.8
2
2.1
2.2
2.3
2.5
2.6
2.7
2.8
2.9
3
3.2
```

3e) <u>Program</u>

```
10  PRINT "ARRANGING NUMBERS IN ASCENDING ORDER":PRINT
20  INPUT "HOW MANY NUMBERS DO YOU WANT TO SORT";A
30  PRINT:PRINT "ENTER THE"; A; "NUMBERS."
40  DIM N(A)
50  FOR L= 1 TO A
60  INPUT N(L)
70  NEXT L
80  P=1
90  FOR L=1 TO A-P
100 IF N(L)<=N(L+1) THEN 120
110 T=N(L): N(L)=N(L+1): N(L+1)=T
120 NEXT L
130 P=P+1
140 IF P>A-1 THEN 160
150 GOTO 90
160 PRINT:PRINT "THE LIST IN ASCENDING ORDER IS ..."
170 FOR L=1 TO A
180 PRINT N(L)
190 NEXT L
200 END
```

Lesson: 61L

AIM: How can we find the median of a set of numbers?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the median of a set of numbers;
2. plan (flowchart) a computer solution for finding the median of a set of numbers;
3. code a program;
4. enter, run, and debug the program.

VOCABULARY: median, mean, average, score

MOTIVATION: Renato received the following grades in Computer Mathematics: 85, 70, 73, 80, 82. If the grades are arranged in ascending order, what is the middle score?

DEVELOPMENT:

1a) After eliciting responses to the motivation problem, say to the students that the middle score for a set of numbers is called the median. Write the definition on the chalkboard. The median of a set of numbers is the middle number when the numbers are arranged in order of size.

1b) Suppose that Renato receives another grade and it is 76. Ask, "What is the median?" Say to the students that since there are an even number of scores, the median is determined by finding the mean of the two middle scores.

The median is $\frac{76+80}{2} = \frac{156}{2} = 78$.

1c) Ask the students to complete the Applications examples.

2. Present the problem which follows for a computer solution.

Problem: Have the computer find the median for A numbers where A, the amount of numbers, is the user's choice.

3. Develop the analysis below with the students.

Analysis

Let us assume that our original list of numbers is stored in N(1), N(2), N(3), ..., N(A).

a) Ask, "Once the **A** numbers have been arranged in ascending order, how do we have the computer find the median if **A** is an odd number?" Let us look at the examples we just did. Ask, "Which number in the list is the middle number?" Record the responses as shown below.

Example	Amount of Numbers in the List	Position of the Middle Number in the List
1	5	3 or (5+1)/2
3	7	4 or (7+1)/2
5	3	2 or (3+1)/2

Say to the students that generally the position of the number in the middle can be found using (A+1)/2. Therefore, the median score would be found in N((A+1)/2).

b) Ask, "Once the **A** numbers have been arranged in ascending order, how do we have the computer find the median if **A** is an even number?" Let us look at the examples we just did. Ask, "How do we find the position of the two middle numbers in the list?" Record the responses as shown below.

Example	Amount of Numbers in the List	Position of the Smaller "Middle" Number	Position of the Next "Middle" Number
Renato's Grades	6	3 or 6/2	4 or 6/2+1
7	10	5 or 10/2	6 or 10/2+1
8	8	4 or 8/2	5 or 8/2+1

Say to the students that generally the position of the numbers in the "middle" can be found using A/2 and A/2+1. Therefore, the median score would be determined by finding the mean of the numbers found in N(A/2) and N(A/2+1); e.g., (N(A/2)+N(A/2+1))/2.

4. Develop the plan which follows with the students.

Plan

1) Put in the amount of numbers, **A**.

2) Enter the numbers, storing them in N(1), N(2), N(3), ..., N(A).

3) Use N(L) to represent a particular number in the list.

4) Use N(L+1) to represent the next number in the list.

5) Ask, "If there are **A** numbers in the list, how many passes will the computer need to make?" (A-1)
Initialize a counter to keep track of the A-1 passes: P=1.

6) Ask, "If there are **A** numbers stored, and memory cell P stores the number for each pass, how do we represent the number of comparisons made during that pass?" (A-P)
Set up a FOR/NEXT loop to move down the list making the comparisons between a number and the next number:
FOR L=L TO $\underbrace{\text{A-P}}$

For each pass, we do not use the memory cell position that will contain the latest number which is arranged in the proper order.

7) Check: Is $N(L) \leq N(L+1)$?
   a) If yes, then go to 9.
   b) If no, continue.

8) Interchange the contents of N(L) and N(L+1).

9) NEXT L.

10) Step up to the next pass: P=P+1.

11) Check: Is P > A-1?
   a) If yes, go to 13.
   b) If no, continue.

12) Go to 6.

13) Check: Is N even?
   a) If yes, then locate the two middle numbers and calculate their mean: MD = (N(A/2) + N(A/2+1))/2.
   Then go to 15.
   b) If no, continue.

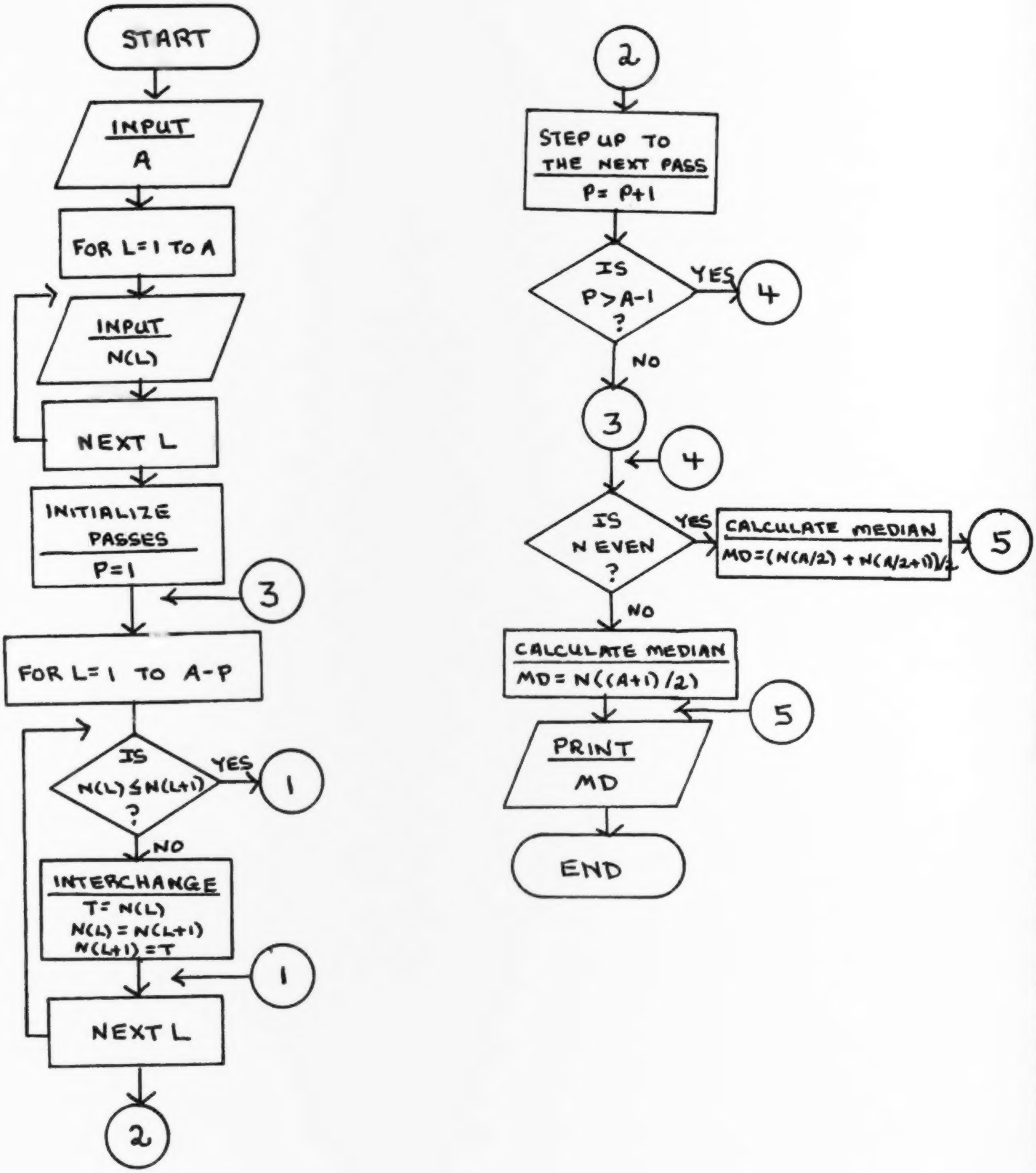
14) Locate the median: MD = N((A+1)/2).

15) Print the median, MD.

16) End

5. Ask the students to develop the flowchart which follows.

Flowchart


START
INPUT
A
FOR L=1 TO A
INPUT
N(L)
NEXT L
INITIALIZE PASSES
P=1
3
FOR L=1 TO A-P
IS N(L)≤N(L+1) ?
YES
1
NO
INTERCHANGE
T= N(L)
N(L) = N(L+1)
N(L+1) =T
1
NEXT L
2
2
STEP UP TO THE NEXT PASS
P= P+1
IS P>A-1 ?
YES
4
NO
3
4
IS N EVEN ?
YES
CALCULATE MEDIAN
MD=(N(A/2) + N(A/2+1))/2
5
NO
CALCULATE MEDIAN
MD= N((A+1)/2)
5
PRINT
MD
END

6a) Ask the students to code the program.

6b) Place the planned output which follows on the chalkboard.

Planned Output

```
FINDING THE MEDIAN
(Skip 1 line.)
HOW MANY #S? 12
ENTER THE 12 NUMBERS.
? 7
? 7.1
? 8
? 7.5
? 8.2
? 7.7
? 8.5
? 7.9
? 8.8
? 9
? 7.3
? 8.1
THE MEDIAN IS 7.95
RUN
FINDING THE MEDIAN
(Skip 1 line.)
HOW MANY #S? 3
ENTER THE 3 NUMBERS.
? 3.2
? 8.7
? 1.4
THE MEDIAN IS 3.2
```

6c) Say, "Notice that we want the computer to be able to use memory cells N(11) and N(12) for additional data." Ask, "How do we signal the computer that we need the extra memory space?" Elicit that once the amount of numbers, A, has been entered by the user, we must include the statement DIM N(A) in our program.

6d) Circulate around the laboratory assisting the students where necessary.

6e) Program

```
10  PRINT "FINDING THE MEDIAN":PRINT
20  INPUT "HOW MANY #'S";A
30  DIM N(A)
40  PRINT "ENTER THE"; A; "NUMBERS."
50  FOR L = 1 TO A
60  INPUT (N(L)
70  NEXT L
80  P=1
90  FOR L=1 TO A-P
100 IF N(L)<=N(L+1) THEN 120
110 T= N(L): N(L)=N(L+1): N(L+1)=T
120 NEXT L
130 P=P+1
140 IF P > A-1 THEN 160
150 GOTO 90
160 IF INT(N/2)*2=N THEN MD=(N(A/2)+N(A/2+1))/2:GOTO 180
170 MD=N((A+1)/2)
180 PRINT "THE MEDIAN IS"; MD
190 END
```

APPLICATIONS:

Find the median for each set of data.

1) 1, 2, 5, 3, 4
2) 2, 9, 2, 9, 7
3) 3, 8, 12, 7, 1, 0, 4
4) 80, 83, 97, 79, 25
5) 3.2, 8.7, 1.4
6) 2, .2, 2.2, .02, 2.02
7) 21, 24, 23, 22, 20, 24, 23, 21, 22, 23
8) 5, 7, 9, 3, 8, 7, 5, 6

SUMMARY:

1. What is the mean of the Computer Mathematics grades considered at the beginning of this lesson: 85, 70, 73, 80, 82?
2. If 85 was replaced by 100, would the mean change? ... the median change?
3. If 70 was replaced by 40, would the mean change? ... the median change?

4. What are the advantages of using the median as a representative score for a set of data?
(Elicit that it can be easily found and that it presents a clearer picture of the data when there are extreme scores which may distort the data's true significance.)

HOMEWORK:

1. Revise the program written in today's lesson so that, in addition to finding the median, the computer also finds the mean.

2. Find the median and the mean for each set of data. In your judgement, is the median a more representative score for each set of data? Why or why not?
   a) 82, 82, 85, 55
   b) 11, 8, 3, 9, 7, 5, 13
   c) \$20, \$15, \$15, \$15, \$100
   d) 75, 70, 80, 85, 85, 85, 75, 75, 80, 80

HOMEWORK SOLUTIONS:

1. The following revisions should be made in the program developed in this lesson.

```
10  PRINT "FINDING THE MEDIAN AND MEAN":PRINT
35  S=0
65  S=S+N(L)
75  MN=S/A
185 PRINT "THE MEAN IS";MN
```

2a) Median = 82, Mean = 76
2b) Median = 8, Mean = 8
2c) Median = \$15, Mean = \$33
2d) Median = 80, Mean = 79